

# Examination of the directions of spillover effects between the real estate and stock prices in Poland using wavelet analysis

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### **Abstract**

The main purpose of this article is to discuss the existence of the spillover effects between the direct real estate, indirect real estate (real estate company stocks) and stock market price indices in Poland and to verify whether the directions of these relationships are stable in the time and frequency domains. For this purpose we have used wavelet analysis methods, such as wavelet coherency and wavelet phase difference analyses, which allow to check the indicated stability and detecting dates of any potential structural changes. The results have not confirmed the hypothesis that the directions of these relationships were constant in the examined period of 2004 Q4 – 2014 Q4, which indicates the existence of structural changes. This means that diffusion scenarios of crises opposite to the 2007–2008 global crisis scenarios, when local property pricing collapses resulted in the global financial crisis, would also be possible. Consequently, the indicated results disprove the usefulness of the knowledge on the current directions of these effects in terms of projecting and/or monitoring the long-term macroprudential or long-term housing policies in Poland, and also give some new insights into investment policy issues.

**Keywords:** real estate market, real estate company stocks, spillover effects

**JEL:** E30, G10, G14, R30

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#### 1. Introduction

The starting point of our analysis is the recent work of Miller et al. (2015) on the time stability of the spillover effect direction between the housing market and the stock market in the USA in the period 1890–2012. The authors have found that on the US markets the credit-price effect, when the changes in real estate returns cause changes in stock market returns, was recurrently replaced by the opposite wealth effect (Kapopoulos, Siokis 2005), which was potentially caused by some structural changes on the above-mentioned markets. These findings have been based on the wavelet analysis methods, derived from the physics methodology, such as the wavelet coherency and the wavelet phase-difference. Such methods are contemporarily used also by economists in order to simultaneously examine changes in terms of co-movement strength, causality sign and causality direction between some economic phenomena in the time and frequency domains (Mandler, Scharnagl 2014; Scharnagl 2011; Soares, Aguiar-Conraria 2011a–c; 2014).

The relations between real estate and stock markets prices were previously examined mainly using traditional methodology of econometrics, such as Granger causality tests (Miller et al. 2015), despite the fact that this method does not provide an opportunity to check the time stability of the direction under consideration and also requires an examination of only one investment horizon at once. Consequently, such methods are strictly dominated by the wavelet analysis methods.

We have examined three research hypotheses:

- 1. There exists a stable direction of the domestic spillover effect between the levels of the rates of return on the price indices of the direct real estate market and stock market in Poland, indicating the stable existence of the wealth effect or the credit-price effect, when the pricing change impulses are constantly originated on the stock market or on the real estate market, respectively.
- 2. There exists a stable direction of the domestic spillover effect between the levels of the rates of return on the price indices of the direct real estate and indirect real estate (real estate company stocks) markets in Poland.
- 3. There exists a stable direction of the domestic spillover effect between the levels of the rates of return on the price indices of the indirect real estate (real estate company stocks) and stock markets in Poland.

Knowledge on the stable direction of the first effect would be useful in terms of conducting macroprudential policy and long-term housing policy in Poland. In particular, in the scope of macroprudential policy it would allow to predict the consequences of potential economic shocks, such as economic crises, due to the presence of the potential constant sequence of property markets and financial markets price diffusion, including their impact on economic growth and financial system stability (Łaszek 2006; Główka 2012). Knowledge on the stable direction of the second and third effects would be potentially useful in terms of conducting investment policy, especially in the light of the ongoing process of the development of the real estate company stocks (indirect property) market as a separate asset class in Poland.

Such research has not yet been conducted using the same methodology in the case of national economies other than the US in the scope of the first of the indicated spillover effects and has not yet been performed at all in the scope of the last two effects. Furthermore, the article contributes to the literature by delivering results of the examination of the time stability of the domination of informational efficiency between the major domestic investment markets in Poland.

#### 2. Literature review

The results of the previous studies on the long-term relationships between the indices under consideration have referred almost solely to some developed economies. These studies have particularly aimed to examine the direction of the causal relationships between the rates of return on price indices of real estate and stock markets, especially studies on the existence of wealth or credit-price effects. Literature on causality directions between the markets under consideration referred especially to the results obtained using Granger causality tests, vector autoregressive models, vector error correction models and threshold error correction models, delivering often mutually exclusive results (Myer, Webb 1993; Okunev, Wilson, Zurbruegg 2000; Lean 2012; Tsai, Lee, Chiang 2012; Hoesli, Oikarinen, Serrano 2015). Such relationships have not yet been examined in the case of Poland, due to the informational inefficiency of the domestic property market, which results in the lack of commercial property indices.

On the basis of the literature review, it should be noted that the existence of such discrepancies could be explained by the differences in frequencies of measurement, selection and methodology of used market price indices, taking into account scopes in time and place dimensions and qualitative time changes in terms of the composition of market portfolios. Such discrepancies could be triggered by the relatively constant occurrence of the flows of capital into the specialized real estate investment funds – REIT markets, which have caused their split as a separate asset class, as well as by the existence of some structural changes between the investment markets under consideration.

# 3. Methodology

The traditional econometric models, as well as traditional spectral Fourier analysis, do not provide tools to analyse economic phenomena in both the time and frequency domains simultaneously.

The econometric methods, including Granger causality direction tests, require the creation of a time constant model (except for in the case of the estimation of the time-varying parameter VAR models) in the form of one or more equations, using at once only one data time interval. In turn, the Fourier transform does not require the creation of a model and provides an opportunity to analyse simultaneously different data intervals. However, its use results in the loss of the time domain information, which makes identification of the dates of transient changes impossible. Therefore, these methods are not adequate to analyse data in terms of recurring structural changes, which may cause temporal reversals of causality directions between economic phenomena. Furthermore, traditional Fourier analysis refers only to stationary time series, whereas the price indices analysed in this article, as well as a large number of macroeconomic time series, exhibit non-stationarity. As a result, in this article we aim to analyse the stability of the relationships under consideration in the time and frequency domains using the wavelet analysis tools, which are commonly used, inter alia, in signal processing and geophysics applications. The toolbox contributed by Aguiar-Conraria and Soares – ASToolbox (Soares, Aguiar-Conraria 2014) has been used to perform simultaneous analysis of the analysed time series in both the time and the frequency domains.

The further part of the "Methodology" section describes the continuous wavelet transform (CWT), which enables to extract self-similarities of data sets, and is based on Miller et al. (2015), Soares,

Aguiar-Conraria, Azevedo (2008), Soares, Aguiar-Conraria (2011b; 2014), Grinsted, Moore, Jevrejeva (2004) and Reboredo and Rivera-Castro (2012). The discrete wavelet transforms, such as the maximum overlap discrete wavelet transform that enable to perform noise reduction and to compress data sets under consideration, are another approach to solving the problem, but require more computational time than the CWT.

Contrary to the infinite duration sine and cosine wave functions used in the Fourier transform, a periodic mother wavelet function is localized in the time and frequency domains finite duration wave function  $\Psi(t) \in L^2(R)$  where t denotes time, which satisfies the so called admissibility condition:

$$\int_{-\infty}^{\infty} \Psi(t) dt = 0 \tag{1}$$

Function  $\Psi(t)$  is subsequently transformed using a translation (or shift) ( $\tau$ ) and a scaling (or dilating) ( $\tau$ ) constant parameters in order to obtain a proper time location and a proper width, respectively:

$$\Psi(t) = \frac{1}{\sqrt{|s|}} \Psi\left(\frac{t-\tau}{s}\right) \tag{2}$$

Larger windows are obtained when |s| > 1 and narrower when |s| < 1.

In this study it has been assumed that the wavelet function is given by the Morlet function, which is often used in economic applications, and the angular Fourier frequency  $\omega_0$  has been set to 6 in order to facilitate the frequency – period reversion analysis (Soares, Aguiar-Conraria 2014):

$$\Psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}$$
(3)

The continuous wavelet transform of a time series denoted by x(t), using  $\Psi(t) \in L^2(R)$  is delivered as:

$$W(\tau,s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} x(t) \Psi\left(\frac{t-\tau}{s}\right) dt, \ \tau,s \in R \land s \neq 0$$
(4)

In turn, the wavelet power spectrum (WPS), describing localized variance of a time series is defined as:

$$WPS(\tau, s) = |WPS(\tau, s)|^{2}$$
(5)

The wavelet phase angle is obtained (in case of a complex valued  $\Psi(t)$  functions) as:

$$\varphi_{x}(\tau,s) = \tan^{-1} \frac{J(W_{x}(\tau,s))}{R(W_{x}(\tau,s))}$$
(6)

where J and R refer to the imaginary and the real parts, respectively.

In order to analyse the relationship between two time series x(t) and y(t) in both the time and frequency domains, we introduce a cross wavelet transform (XWT) as:

$$W_{vv}(\tau,s) = W_{v}(\tau,s) \overline{W_{v}(\tau,s)}$$
(7)

where  $W_x(\tau, s)$  and  $W_y(\tau, s)$  are CWTs of an x(t) function and a y(t) function, respectively.

The cross wavelet power (XWP), describing a localized covariance between two time series x(t) and y(t), is introduced as:

$$XWP_{xy}(\tau,s) = \left| W_{xy}(\tau,s) \right| \tag{8}$$

The squared wavelet coherency gives information on time and frequency localized comovements (correlations) between a pair of analysed time series and is defined as:

$$R_{xy}^{2}(\tau,s) = \frac{\left| S(s^{-1}W_{xy}(\tau,s)) \right|^{2}}{S(s^{-1}|W_{x}(\tau,s)|^{2})S(s^{-1}|W_{y}(\tau,s)|^{2})}$$
(9)

where *S* is a smoothing operator and:

$$0 \le R_{xy}(\tau, s) \le 1 \tag{10}$$

where 0 indicates low correlation and 1 indicates high correlation.

The wavelet phase difference, the outline of which is depicted in the first diagram, provides information on time changes of causality direction (phase lead of x(t) over y(t)), as well as information on time changes of the sign of comovements between two analysed time series and is defined as:

$$\varphi_{x}(\tau,s) = \tan^{-1} \frac{J(W_{xy}(\tau,s))}{R(W_{xy}(\tau,s))} \qquad \varphi_{x}(\tau,s) \in \langle -\pi,\pi \rangle$$
(11)

or:

$$\varphi_{xy}(\tau,s) = \varphi_x(\tau,s) - \varphi_y(\tau,s) \tag{12}$$

According to Aguiar-Conraria and Soares phase difference test results are significant only when wavelet coherency test results are statistically significant in both cases of considered time and considered frequency band (Soares, Aguiar-Conraria 2014).

#### 4. Data

Our study focuses on the price movements on the three kinds of markets: the direct and indirect property markets, as well as the stock market in Poland in the period starting on 31 December 2004 and finishing on 31 December 2014.

We consider the three time series of the standardised logarithmic rates of return on the price indices, previously deflated using the consumer price index estimated by the Polish Central Statistical Office as of 31 December 2004, encompassing the period 2004 Q4–2014 Q4. The sequence of the data transformations has been performed in line with the methodology used by Miller et al. (2015) to allow for international comparison and to correct for any potential heteroscedasticity. However, the wavelet analysis also enables to perform analogical research on price indices due to its ability to examine stationary as well as only locally stationary time series (Bruzda 2013; Miller et al. 2015).

The list of the considered market price indices, which are presented in the Figure 1, is as follows:

- 1. Price index of real estate companies (the Polish indirect property market price index), retraced using own estimations in accordance with the Warsaw Stock Exchange indices rules to replicate the WIG-developers index, which was not estimated for the period before the 2007 Q2.
- 2. The investment property fair value index (the latent Polish direct property market price index, covering especially the commercial real estate markets, based on the sector level appraisal data available since 2005 when the International Accounting Standard no. 40 Investment property came into force, enabling the use of an appraisal-based valuations approach instead of a historical cost approach), according to own estimations based on the cyclical revaluations of the aggregated investment portfolios held from the beginning of successive periods, disclosed in the quarterly financial statements (done in accordance with the IAS no. 40) of the enterprises included in the WIG-developers portfolio as of the 31 December 2014, mirroring to some extent the stock market price indices estimation rules, including not taking into account the disposal gains (extensively explained in Koltuniak 2016).
- 3. The WIG index (the Polish overall stock market price index), according to the Warsaw Stock Exchange data.

According to the results published in Kołtuniak (2016), indicated price indices were cointegrated in the period under consideration, which means that the long-term equilibrium reversion tendencies between them were present, which legitimatise the check on their causality directions.

Despite the fact that quarterly data have been used due to the lack of higher frequency data connected with the real estate markets, the wavelet analyses, which are often used to analyse high frequency data sets, have been performed with the aim of detecting structural changes that potentially influenced the directions of spillover effects under consideration.

#### 5. Results

Table 1 depicts the results of the augmented Dickey-Fuller tests on the non-stationarity of the considered time series. These results indicate that all of the considered time series are integrated of order 1, which legitimatizes the use of the Granger causality tests. The results of the examination of the considered spillover effects using Granger causality direction tests are depicted in Table 2.

These results indicate that in the period between 2005 and 2014 pricing change impulses on the stock market in Poland originated on the real estate market.

The existence of the credit-price effect has been confirmed only in the case of the direct property and indirect property markets causality and has not been confirmed in the case of the direct property and stock markets causality. The existence of the wealth effect has been denied in the case of both the direct and indirect property markets.

Figures 2–7 depict the results of the wavelet analyses performed in order to verify the research hypotheses, using both the squared wavelet coherency and the wavelet phase-difference methods.

The thick black lines in Figures 2, 4 and 6 cut the regions in which exist the edge effects connected with supplementing short data sets with zeros. The black thin lines in Figures 2, 4 and 6 designate the 5% level of significance, estimated using the Monte Carlo simulations. These lines indicate also time periods in which phase difference functions presented in Figures 3, 5 and 7 were statistically significant within the frequency bands, which were previously designated in line with the indicated significance lines (Soares, Aguiar-Conraria 2014).

In the case of the scales of Figures 2, 4, and 6, the bright and dark colours designate low and high squared coherency (correlation) between the analysed time series, respectively. The indicated figures do not allow to distinguish between negative and positive correlations.

Figures 2 and 3 verify the first research hypothesis on the stable existence of the wealth or the credit-price effects in Poland in the period 2004 Q4 – 2014 Q4. The returns on the direct real estate and on the stock market price indices did not display significant comovements in the period before and during the world crises on the real estate and financial markets (2005–2008). In the period 2009–2014, the comovements became significant for the frequency band between half a year and 1.5 years. Consequently, for this period the phase differences in the frequency bands 0.5–1 and 1–1.5 years have been checked. Figure 3 shows that in the indicated period, as well as in the entire sample period, the credit-price effect was replaced by the wealth-effect and vice versa multiple times. Consequently, the relationship direction is not stable in time. As a result, the first research hypothesis has been rejected. This result is in line with the findings for the US markets published by Miller et al. (2015).

Figures 4 and 5 verify the second research hypothesis on the stable existence of the sectorial spillover effect direction (Rua, Nunes 2009) between the logarithmic rates of return on the price indices of the direct and indirect property markets in Poland in the period 2004 Q4 - 2014 Q4. In the periods 2004 Q4 – 2007 Q2 and 2008 Q3 – 2013 Q1 indicated rates of return displayed statistically significant high comovements (correlations) in the frequency band between half and 1.5 years. During the real estate and the global financial crises these series displayed high correlations in the frequency band near 2 years. After 2013 Q2, no statistically significant comovements have been displayed. Consequently, for the period till 2013 Q2, the phase differences in the frequency bands 0.5–1 and 1–2 years have been checked. Figure 5 shows that the changes in the level of the indirect property market price index returns led the changes in the level of the direct property market index returns, only in the periods 2004 Q4 - 2007 Q1 and 2012 Q2 - 2014 Q4, which is in line with the hypothesis on the efficient capital markets. In the vast majority of the entire period under consideration the changes in the level of the direct property market price index returns led the changes in the level of the indirect property market price index returns, which indicates the existence of some inefficiencies in the Polish real estate company stocks market. Consequently, the second research hypothesis has also been rejected.

Figures 6 and 7 verify the last research hypothesis on the stable existence of the direction of the spillover effect between the logarithmic rates of return on the price indices of the indirect property market and entire capital market. In the period 2005 – 2006 Q2 the indicated rates of return displayed statistically significant high comovements (correlations) in the relatively high frequency band between 0.25 and 0.75 years and in the period 2008–2012 in the frequency band between 0.5 and 2 years, as well as in the period 2007–2012 in the frequency band 2.5–6 years. After 2013 Q2 no statistically significant comovements have been displayed. Consequently, for the period till 2013 Q2 the phase differences in the frequency bands 0.25–1.5 and 2–5 years have been checked. Figure 7 shows that the changes in the level of the indirect property market price index returns led the changes in the level of the entire stock market price index returns in the period 2004 Q4 – 2011 Q4 and then the changes in the level of the entire stock market price index returns led the changes in the level of the real estate company stock market price index returns. Consequently, the last research hypothesis has also been rejected.

## 6. Conclusions and policy implications

Rejection of the first research hypothesis on the stability of the direction of the causal relationship between the real estate and stocks prices in Poland indicates that knowledge on the current direction of this causality would not be permanently viable, which invalidates its use for making assumptions in the scope of projecting or monitoring the long-term macroprudential policy and the long-term housing policy in Poland, as well as in terms of investment policy and in the scope of predicting overall economic growth rates or directions of the diffusion of crises.

Rejection of the first research hypothesis is in line with the recently published article using analogical methodology by Miller et al. (2015) in the case of the yearly rates of return on the developed US housing market and stock markets price indices in 1890–2012.

Contrary to the research by Miller et al., the presented study also covered the comovements and causalities between the direct and indirect property markets, as well as between the indirect property market and stock market in Poland.

Rejection of the second research hypothesis indicates that there is no stability in terms of the informational efficiency domination of the one over the other property markets in Poland, which means that in the long term there is no possibility to anticipate price changes on these markets by constantly using one of the real estate price indices as a predictor. Rejection of the third research hypothesis brings analogical conclusions in terms of the indirect property market and entire stock market in Poland.

The instabilities under consideration may have been caused by any structural changes on the direct and indirect property markets, as well as in the shape of the real estate financing system in Poland.

Phase-difference analyses between all of the pairs of time series under consideration have confirmed the constant presence of positive signs of the comovements between them, which is in line with the economic theory and influences investment decision processes. Squared wavelet coherency analyses have revealed that the statistically significant comovements between the indicated pairs proceeded mainly in the mid and long term. The results of the phase-difference tests have also confirmed that the direction of any of the considered economic spillover effects would depend on a frequency or on a period assumed during the estimation of examined rates of return.

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# **Appendix**

Table 1
Results of the augmented Dickey-Fuller tests in the sample period 2004 Q4 – 2014 Q4

Variable	Time series in levels			First differences of natural logarithms of the time series				
	test with a constant	test with a constant and a linear trend	test with a constant and a linear and quadratic trends	lags number	test with a constant	test with a constant and a linear trend	test with a constant and a linear and quadratic trends	lags number
	test statistic (p-value)	test statistic (p-value)	test statistic (p-value)		test statistic (p-value)	test statistic (p-value)	test statistic (p-value)	
Real estate company stock index	-1.08* (0.72)	-2.02* (0.57)	-1 .66* (0.90)	7	-4.50 (0.00)	-3.43 (0.05)	-4.08 (0.04)	8
Investment property fair value index	-2.46* (0.13)	-1.87* (0.67)	-2.99* (0.30)	1	-5.27 (0.00)	-5.89 (0.00)	-5.31 (0.00)	0
WIG index	-1.97* (0.30)	-1.88* (0.65)	-1.89* (0.84)	0	-3.95 (0.00)	-3.93 (0.02)	-3.97 (0.04)	0

#### Notes:

The results indicate that all of the considered time series have been integrated of order 1, which legitimatizes the use of the Granger causality tests. The results were also confirmed by the KPSS test results.

Sign \* indicates that the 5% significance level of confirmation of the null hypothesis on non-stationarity of the analysed times series was exceeded.

Lags number chosen automatically using AIC.

Real estate company stock index and investment property fair value index according to the own estimations; WIG index according to WSE data.

Table 2
Results of the estimation of the F-statistics of the Granger causality tests

H0 = time series A does not cause time series B					
	Real estate company stock index	Investment property fair value index	WIG index		
Real estate company stock index	×	0.94 (0.46)	2.27 (0.09)		
Investment property fair value index	3.83* (0.01)	×	1.44 (0.25)		
WIG index	0.88 (0.49)	0.44 (0.78)	×		

#### Notes:

The pairs of the first differences of natural logarithms of the price indices listed in the "Data used" section, in the sample period  $2005 \, \text{Q1} - 2014 \, \text{Q4}$  (assuming 4 lags), using quarterly data intervals.

Time series are presented in the following way: series from the set of time series A in the vertical dimension, series from a set of time series B (i.e. replicated series A) in the horizontal dimension.

Sign \* indicates that the 5% significance level of confirmation of the null hypothesis of no causal relationship between analysed times series was not exceeded, i.e. the Granger causality relationship between them was not rejected, however its confirmation requires additional corroboration with the economic theory; p-values in brackets.

Diagram 1
Outline of a wavelet phase difference function

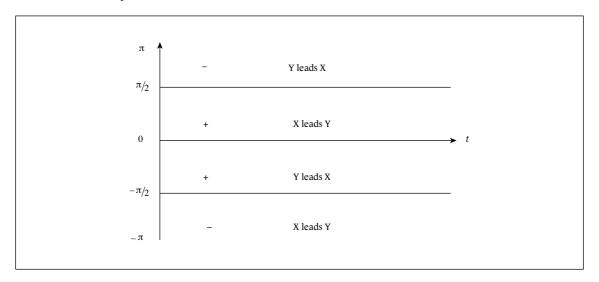
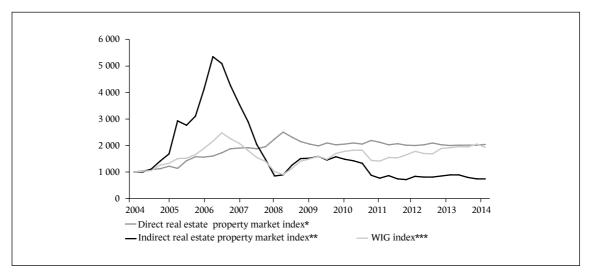


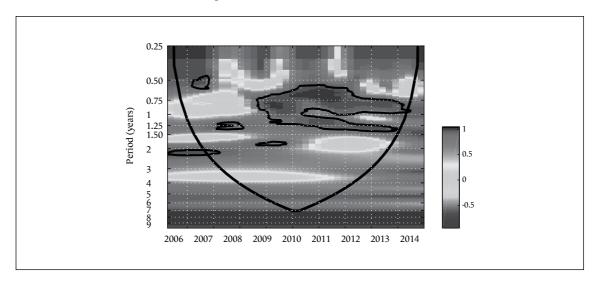
Figure 1 Comparison of the fluctuations in the price indices of the direct real estate, indirect real estate and stock markets in Poland in the period 2004 Q4 - 2014 Q4



<sup>\*</sup> Appraisal based fair value index of the aggregated portfolio of investment property of enterprises being in the scope of the research according to the own estimations.

Note: data normalized to the level of 1000 points as of 31 December 2004.

Figure 2 Wavelet squared coherency between the logarithmic rates of return on the price indices of the direct real estate and the stock markets in Poland in the period  $2004 \, \text{Q4} - 2014 \, \text{Q4}$ 



<sup>\*\*</sup> Price index of the property enterprises stocks being in the scope of the research retraced according to the own estimations.

\*\*\* According to the WSE data.

Figure 3
Wavelet phase-difference between the logarithmic rates of return on the price indices of the direct real estate and the stock markets in Poland in the period 2004 Q4 – 2014 Q4

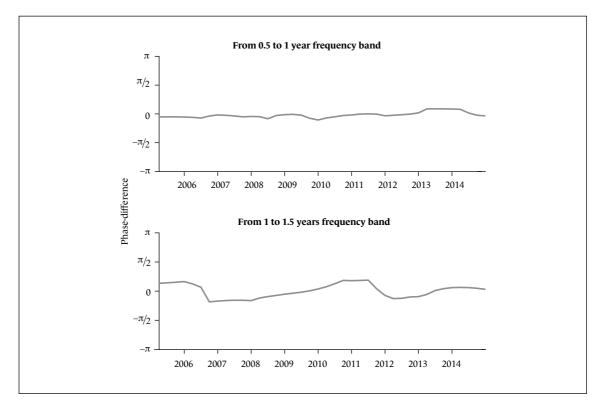


Figure 4 Wavelet squared coherency between the logarithmic rates of return on the price indices of the direct and the indirect real estate markets in Poland in the period 2004 Q4 - 2014 Q4

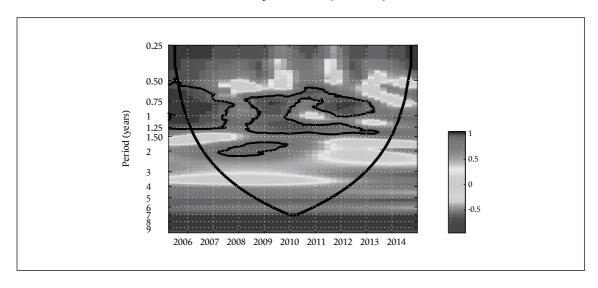


Figure 5 Wavelet phase-difference between the logarithmic rates of return on the price indices of the direct and the indirect real estate markets in Poland in the period 2004 Q4 - 2014 Q4

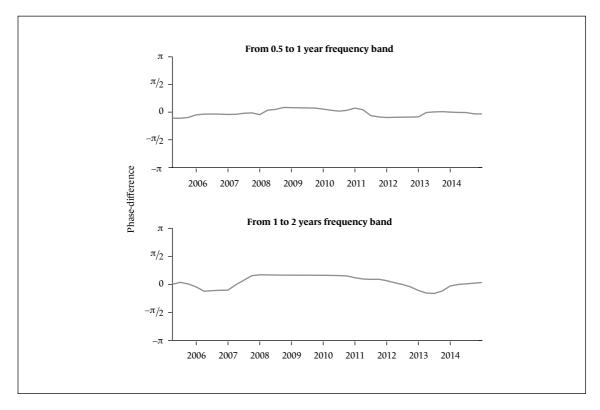


Figure 6
Wavelet squared coherency between the logarithmic rates of return on the price indices of the indirect real estate market and the stock market in Poland

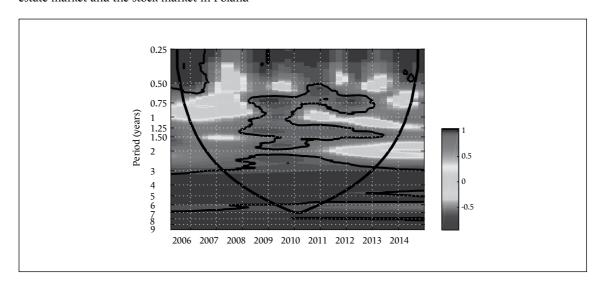


Figure 7 Wavelet phase-difference between the logarithmic rates of return on the price indices of the indirect real estate and the stock markets in Poland in the period  $2004 \, \text{Q4} - 2014 \, \text{Q4}$ 

