Bank i Kredyt 45(6), 2014, 513–536

BANK I KREDYT
BANK & CREDIT

Is illiquidity risk priced? The case of the Polish medium-size emerging stock market

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Submitted: 12 December 2013. Accepted: 8 October 2014.

Abstract

This paper explicitly tests the hypothesis that illiquidity risk is not priced in the Polish medium-size emerging stock market. To address this issue, we employ a liquidity-adjusted capital asset pricing model which explains how asset prices are affected by illiquidity risk and commonality in liquidity. The model takes into consideration various sources of illiquidity risk. In contrast to previous studies for the U.S. developed stock market, our empirical results indicate no reason to reject the research hypothesis that illiquidity risk is not priced in the Warsaw Stock Exchange.

Keywords: asset pricing, illiquidity risk, commonality in liquidity, LCAPM, Polish stock market

JEL: C32, C58, G12

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1. Introduction

The classic theory of finance is based on the assumption of a frictionless, perfectly liquid market, where every security can be traded at no cost all of the time, and agents take prices as given (Amihud, Mendelson, Pedersen 2005). Cochrane (2005b) stressed that for a long time, there has been an implicit separation of effort in asset pricing. The majority of researchers operated in the frictionless macroeconomics-based tradition, while other researchers investigated the area concerning asset pricing models including market microstructure effects. Recently, this separation has begun to erode. This is mainly due to a growing understanding of the crucial role of liquidity, trading, volume, bid/ask spread, other transaction costs, etc. Bekaert, Harvey and Lundblad (2007) among others pointed out that liquidity is particularly important for asset pricing. Illiquid assets and assets with high transaction costs trade at low prices relative to their expected cash flows. For this reason, quite many studies empirically tested the hypothesis that illiquidity risk is priced (e.g. Amihud, Mendelson 1986; Brennan, Subrahmanyam 1996; Datar, Naik, Radcliffe 1998; Pastor, Stambaugh 2003; Acharya, Pedersen 2005; Martinez et al. 2005; Chen 2005; Liu 2006; Sadka 2006; Korajczyk, Sadka 2008; Lee 2011; Olbryś 2014).

It is worthwhile to note that the potential importance of liquidity/illiquidity has not been explored as extensively in international stock markets (especially, in emerging markets) as in the U.S. market – arguably, the most liquid market in the world.[1] Especially popular in the literature are various multifactor "classical" models, which incorporate a liquidity risk factor into an asset pricing relationship:

1) the liquidity-augmented capital asset pricing model,
2) the liquidity-augmented Fama-French model (1993),
3) the liquidity-augmented Carhart's model (1997).

As for the U.S. stock market, we should mention the papers: Brennan, Chordia, Subrahmanyam (1998); Datar, Naik, Radcliffe (1998); Chordia, Subrahmanyam, Anshuman (2001), Avramov, Chordia (2006); Liu (2006). In contrast, relatively few papers focused on the other financial markets. For example, Chan and Faff (2005) examined the asset pricing role of liquidity for the Australian stock market. Miralles Marcelo and Miralles Quirós (2006) applied the liquidity-augmented Fama-French model (1993) for the Spanish stock market. Martinez et al. (2005) employed either unconditional or conditional versions of liquidity-based asset pricing models in the Spanish market. Chang, Faff and Hwang (2010) tested the liquidity-augmented Fama-French model (1993) in the Tokyo Stock Exchange. Lischewski and Voronkova (2012) used the classical liquidity-augmented multifactor asset pricing models in the case of the Warsaw Stock Exchange (WSE). They investigated whether the factors that were found to be important for the developed and other emerging markets also played a role in asset pricing in the Polish market. It is worthwhile to note that our methodology substantially differs from the Lischewski and Voronkova (2012) approach, as we employ the "non-classical" asset pricing model.

Some "non-classical" asset pricing models appeared in the literature over recent years. One of the first studies which investigated the role of illiquidity in asset pricing was (Amihud, Mendelson 1986). The authors formalized the important link between the bid/ask spread and asset returns. They showed that, in equilibrium, illiquid assets would be held by investors with longer investment horizons. Amihud and Mendelson suggested that an investor with a long investment horizon, compared to

[1] For comparison, annual share trading value in 2013 was equal to: (1) USD 25,722 billion for Americans, (2) USD 19,887 billion for Asia-Pacific, and (3) USD 9,092 billion for Europe, Africa and Middle East. For the NYSE Euronext (U.S.) it was equal to USD 13,700 billion (source: http://www.world-exchanges.org).

an investor with a shorter horizon, would require a smaller premium for illiquidity as reflected in the bid/ask spread. Another interesting proposition was the model presented in the paper (Jacoby, Fowler, Gottesman 2000). The authors developed a liquidity-adjusted version of the capital asset pricing model based on the returns calculated after taking into account the effect of the bid/ask spread. They advocated that the measure of systematic risk should incorporate illiquidity costs (i.e., the bid/ask spread).

However, the most general, compound, and empirically complicated, out of the other liquidity--adjusted asset pricing models, is the model presented in the paper (Acharya, Pedersen 2005). This non-classical and liquidity-adjusted version of the CAPM (LCAPM) incorporates four channels for an illiquidity premium (Cochrane 2005b). As the purpose of this paper is to examine the LCAPM in the Polish stock market, the LCAPM will be presented in details in Section 3. In brief, Acharya and Pedersen (2005) argued that liquidity is risky and varies over time both for individual stocks and for the market as a whole. Their theoretical model helps explain how asset prices are affected by illiquidity risk and commonality in liquidity. Commonality in liquidity refers to the impact of a common or a market-wide liquidity factor on an individual security (Brockman, Chung, Perignon 2009). It is worth stressing that commonality in liquidity was the centre of attention in many empirical research papers (e.g. Chordia, Roll, Subrahmanyam 2000; Hasbrouck, Seppi 2001; Huberman, Halka 2001; Pastor, Stambaugh 2003; Acharya, Pedersen 2005; Martinez et al. 2005; Sadka 2006; Korajczyk, Sadka 2008; Brockman, Chung, Perignon 2009; Lee 2011). To sum up, in the context of asset pricing, if liquidity varies systematically, securities with returns positively correlated with market liquidity should have higher expected returns Bekaert, Harvey, Lundblad (2007).

As a matter of fact, emerging economies are particularly interesting in many respects. Bekaert and Harvey (2002) argued that emerging markets constitute ideal laboratories to test predictions regarding liquidity and asset prices. Liquidity effects may be particularly acute in emerging markets (Bekaert, Harvey 2003). As for the European emerging stock markets, Smith (2009) stressed that these stock markets can be usefully classified in three groups: Russia, four medium-size markets: the Czech Republic, Hungary, Poland, and Turkey, and a group of 19 small, new markets. An extensive survey of recent research on emerging markets within the fields of economics, finance, international business, and management was proposed by Kearney (2012).

The main motivation for our study is provided by the growing interest in liquidity/illiquidity in the context of asset pricing that has emerged in the literature over recent years. The goal of this paper is to explicitly test the hypothesis that illiquidity risk is not priced in the case of the Polish medium-size emerging stock market. To address this issue, we employ a liquidity-adjusted capital asset pricing model (LCAPM), proposed by Acharya and Pedersen (2005). Using the non-classical asset pricing approach is concluded to be purposeful, as Olbryś (2014) found that there were no reasons to reject the research hypothesis that various frictions in trading processes are present in the Warsaw Stock Exchange.

The LCAPM takes into consideration various sources of illiquidity risk. Following Acharya and Pedersen (2005), we apply the Amihud's (2002) measure of illiquidity. In contrast to previous studies for the U.S. stock market, our empirical results reveal no reason to reject the hypothesis that illiquidity risk is not priced in the WSE. To the best of the author's knowledge, no such research has been undertaken for the Polish emerging stock market.

The remainder of this study is organized as follows. Section 2 briefly describes some measures of liquidity/illiquidity. Section 3 specifies a methodological background of theoretical framework concerning the LCAPM (Acharya, Pedersen 2005). In Section 4, we present the research procedure and data for the WSE. In Section 5, we present and discuss the empirical results obtained. Section 6 recalls the main findings and presents the conclusions.

2. Measuring of liquidity/illiquidity in emerging markets

Liquidity, by its very nature, is difficult to define and even more difficult to estimate (Lesmond 2005). Bekaert and Harvey (2003) stressed that obtaining estimates of transaction costs and illiquidity is important because liquidity/illiquidity is probably priced as illiquid assets and assets with high transaction costs trade at low prices, relative to their expected cash flows. It follows that liquidity and trading costs may contribute to the average equity premium. Kyle (1985) argued that market liquidity is a slippery and elusive concept, in part because it encompasses a number of transactional properties of markets. Empirical liquidity definitions span direct trading costs, measured by the bid/ask spread (quoted or effective), to indirect trading costs, measured by price impact (Lesmond 2005). For example, Stoll (2000) presented a brief review of illiquidity measures, i.e. the quoted, effective, and traded spreads, based on the high-frequency intraday transactions data.

However, direct measurement of liquidity, bid/ask spreads, other trading costs, etc. is difficult and even impossible as intraday trading data are not available free of charge in the case of most emerging stock markets. The lack of access to intraday trading data for emerging markets in general is a fact that is both widely known and amply commented in the literature (e.g. Lesmond 2005; Bekaert, Harvey, Lundblad 2007). Given the uncertainty surrounding liquidity estimation, some liquidity/illiquidity measures are especially often advocated in the literature to provide empirical research in liquidity/illiquidity effects in emerging markets.

The popular measures of trading activity, i.e. volume, dollar trading volume, and share or market turnover, are the simplest measures of liquidity. The raw trading volume is the number of shares traded. The stock turnover is defined as the ratio of the number of shares traded in a day to the number of shares outstanding at the end of the day. It is worthwhile to note that using turnover disentangles the effect of firm size from trading volume. The market turnover is the ratio of the shares traded to market capitalization. These relatively simple measures of liquidity were widely used in the literature (e.g. Datar, Naik, Radcliffe 1998; Chordia, Subrahmanyam, Anshuman 2001; Chen 2005; Lesmond 2005; Bekaert, Harvey, Lundblad 2007; Goyenko, Holden, Trzcinka 2009; Lischewski, Voronkova 2012).

Roll (1984) developed a simple implicit measure of the effective bid/ask spread in an efficient market, based on the first-order serial covariance of price changes or security returns. However, Stoll (2000) stressed that the use of serial covariance as a measure of friction assumes that there are no other sources of serial covariance. The Roll's (1984) measure requires a negative serial covariance in the returns or price changes. When the sample serial covariance is positive, the Roll's (1984) formula is undefined. To avoid this problem, Goyenko, Holden and Trzcinka (2009) proposed a modified version of the Roll's (1984) estimator.

Lesmond, Ogden and Trzcinka (1999) introduced two simple, but useful measures of illiquidity, which may be denoted as ZERO1 and ZERO2. The authors found that zero returns were very frequent in the case of the NYSE/AMEX securities from 1963 to 1990. Therefore, they proposed the proportion of days with zero returns as a proxy for illiquidity. Lesmond, Ogden and Trzcinka (1999) defined the proportion of days with zero returns as:

$$\text{ZERO1} = \frac{\text{the number of days with zero return}}{T}$$

$$\text{ZERO2} = \frac{\text{the number of positive-volume days with zero return}}{T}$$

where T is the number of trading days in a month.

The ZERO1 and ZERO2 measures were used by Lesmond (2005); Bekaert, Harvey, Lundblad (2007), Goyenko, Holden, Trzcinka (2009), Chang, Faff, Hwang (2010), Lee (2011). Furthermore, Lesmond, Ogden and Trzcinka (1999) developed a new liquidity estimator, known in the literature under the acronym LOT (Lesmond 2005). The model of security returns in the presence of transaction costs is based on the limited dependent variable model (Maddala 2001). In brief, the LOT liquidity measure is the difference between the percent buying cost and the percent selling cost. This estimator of the effective spread is based on the assumption of informed trading on non-zero-return days and the absence of informed trading on zero-return days (Goyenko, Holden, Trzcinka 2009).

Pastor and Stambaugh (2003) introduced a measure of price impact called Gamma. The Gamma measure of an individual stock is calculated with daily return and volume data within the month, based on the OLS regression. Intuitively, Gamma measures the reverse of the order flow shock of the previous day. Gamma should have a negative sign. The larger the absolute value of Gamma, the larger the price impact (Goyenko, Holden, Trzcinka 2009).

Amihud (2002) developed a price impact measure that captured daily price response associated with one dollar of trading volume. This measure of illiquidity is often denoted as $ILLIQ_t^i$ for asset i in month t:

$$ILLIQ_t^i = \frac{1}{D_t^i} \sum_{d=1}^{D_t^i} \frac{\left|R_{t,d}^i\right|}{V_{t,d}^i} \tag{1}$$

where $R_{t,d}^i$ and $V_{t,d}^i$ are the return and the dollar volume for asset i, on day d, of month t, respectively, and D_t^i is the number of trading days in month t.

The measure is calculated over all positive-volume days, since the ratio (1) is undefined for zero--volume days. The Amihud's (2002) measure of illiquidity (1) was recently widely recommended in the literature, also in the case of emerging stock markets (see e.g. Acharya, Pedersen 2005; Lesmond 2005; Chen 2005; Miralles Marcelo, Miralles Quirós 2006; Bekaert, Harvey, Lundblad 2007; Korajczyk, Sadka 2008; Goyenko, Holden, Trzcinka 2009; Chang, Faff, Hwang 2010; Lischewski, Voronkova 2012).

Acharya and Pedersen (2005) employ the illiquidity measure of Amihud (2002) given by eq. (1) to show that expected stock (portfolio) returns are a function of several terms: first, expected stock (portfolio) illiquidity and, second, some covariances between stock (portfolio) returns, stock (portfolio) illiquidity, market returns, and market illiquidity. Miralles Marcelo and Miralles Quirós (2006) pointed out that the advantage of using the Amihud's (2002) illiquidity measure is twofold. First, it appears to be the best among the proxies employed to capture the Kyle's (1985) theoretical price impact approach. Second, the data on illiquidity rates is relatively easy to obtain.

3. The LCAPM specification

The liquidity-adjusted capital asset pricing model (LCAPM) of Acharya and Pedersen (2005) is derived in a framework similar to the classical CAPM. However, the main advantage of the LCAPM is that it incorporates the trading cost as a random variable into the asset pricing model. In the LCAPM, the trading cost-free stock price is replaced with the price that is adjusted by the stochastic trading cost. As Campbell (2000, pp. 1515–1516) argued: "The starting point for every financial model is the uncertainty facing investors, and the substance of every financial model involves the impact of uncertainty on the behavior of investors and, ultimately, on market prices. The random fluctuations that require the use of statistical theory to estimate and test financial models are intimately related to the uncertainty on which those models are based". In the next part of this section we briefly introduce the LCAPM, based on Acharya and, Pedersen (2005).

The LCAPM assumes a simple overlapping generations economy in which a new generation of agents is born at any time t, $t \in \{\ldots,-2,-1,0,1,2,\ldots\}$. A generation t consists of I_t investors. An investor $n \in I_t$ has an endowment at time t, trades in periods t and $t + 1$, and derives utility from consumption at time $t+1$. He/she has constant absolute risk aversion A^n. There are N securities indexed by $i \in \{1,2,\ldots,N\}$, with a total of S_i shares of security i. At time t security i pays a dividend of $D_{i,t}$, has a share price $P_{i,t}$, and has an illiquidity cost of $K_{i,t}$. The illiquidity cost, $K_{i,t}$, is modeled as the per-share cost of selling security i. Therefore, the agent can buy at $P_{i,t}$ but must sell at $(P_{i,t} - K_{i,t})$. Short-selling is not allowed. The variables $D_{i,t}$ and $K_{i,t}$ are random variables which are defined on a probability space (Ω, F, P), and all random variables indexed by t are measurable with respect to the filtration $(F_t)_{t\in T}$, representing the information commonly available to investors at time t. In the LCAPM, illiquidity risk is connected with uncertainty about illiquidity costs.

The model assumes that investors can borrow and lend at a risk-free real gross return of $R^G_{F,t} = 1 + R_{F,t} > 1$. The purpose of the LCAPM is to explain how the expected (gross) return $R^G_{i,t}$ of an asset i, at time t, given by eq. (2):

$$R^G_{i,t} = 1 + R_{i,t} = \frac{P_{i,t} + D_{i,t}}{P_{i,t-1}} = \frac{X_{i,t}}{P_{i,t-1}} \tag{2}$$

depends on:

1) the relative illiquidity cost $k_{i,t}$ of an asset i at time t:

$$k_{i,t} = \frac{K_{i,t}}{P_{i,t-1}} \tag{3}$$

2) the market index (gross) return $R^G_{M,t}$ at time t:

$$R^G_{M,t} = 1 + R_{M,t} = \frac{\sum_{i=1}^{N} S_i (P_{i,t} + D_{i,t})}{\sum_{i=1}^{N} S_i P_{i,t-1}} = \frac{\sum_{i=1}^{N} S_i X_{i,t}}{\sum_{i=1}^{N} S_i P_{i,t-1}} \tag{4}$$

3) the relative market illiquidity $k_{M,t}$ at time *t*:

$$k_{M,t} = \frac{\sum_{i=1}^{N} S_i K_{i,t}}{\sum_{i=1}^{N} S_i P_{i,t-1}} \tag{5}$$

Hence, in the unique linear equilibrium, the conditional expected net return of a security *i* is given by eq. (6) (Acharya, Pedersen 2005):

$$E_t(R_{i,t+1}^G - k_{i,t+1}) = R_{F,t}^G + \lambda_t \frac{Cov_t(R_{i,t+1}^G - k_{i,t+1}, R_{M,t+1}^G - k_{M,t+1})}{Var_t(R_{M,t+1}^G - k_{M,t+1})} \tag{6}$$

where $\lambda_t = E_t(R_{M,t+1}^G - k_{M,t+1} - R_{F,t}^G)$ is the risk premium.[2]

Equivalently, the conditional expected gross return of a security *i* is given by eq. (7):[3]

$$\begin{aligned} E_t(R_{i,t+1}^G) = R_{F,t}^G + E_t(k_{i,t+1}) + \lambda_t \frac{Cov_t(R_{i,t+1}^G, R_{M,t+1}^G)}{Var_t(\cdot)} + \lambda_t \frac{Cov_t(k_{i,t+1}, k_{M,t+1})}{Var_t(\cdot)} + \\ + \lambda_t \frac{Cov_t(R_{i,t+1}^G, k_{M,t+1})}{Var_t(\cdot)} + \lambda_t \frac{Cov_t(k_{i,t+1}, R_{M,t+1}^G)}{Var_t(\cdot)} \end{aligned} \tag{7}$$

where $Var_t(\cdot) = Var_t(R_{M,t+1}^G - k_{M,t+1})$, like in eq. (6).

The four conditional covariances in eq. (7) represent various sources of risk and each of them has an economic interpretation. To wit:

1. The covariance $Cov_t(R_{i,t+1}^G, R_{M,t+1}^G)$ defines the systematic risk of an asset *i*, by analogy with the classical CAPM.

2. The covariance $Cov_t(k_{i,t+1}, k_{M,t+1})$ represents the first illiquidity effect. This effect consists in that the expected return increases with the covariance between the asset illiquidity and the market illiquidity. The main reason is that investors want to be compensated for holding a security that becomes illiquid when the market in general becomes illiquid. This is the effect of commonality in liquidity, as mentioned in Introduction.

3. The term $Cov_t(R_{i,t+1}^G, k_{M,t+1})$ defines the second illiquidity effect, which is due to the covariation between the security return and the market illiquidity. This covariance affects required returns negatively as investors are willing to accept a lower return on an asset with a high return in times of market illiquidity.

4. The last term $Cov_t(k_{i,t+1}, R_{M,t+1}^G)$ represents the third illiquidity effect, which is due to the covariation between the asset illiquidity and the market return. This effect arises from the willingness of investors to accept a lower expected return on a security that is liquid in a down market. Hence, the fourth covariance affects required returns negatively.

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2 When the relative illiquidity costs (3) and (5) are both equal to zero, the model (6) is equivalent to the classical CAPM.

3 Based on the following theorems for any random variables X, Y: (1) $Cov(X,Y) = E(XY) - E(X)E(Y)$ and (2) $E(X+Y) = E(X) + E(Y)$.

To estimate the LCAPM, Acharya and Pedersen (2005) derive an unconditional version of the model (7), which can be obtained, for instance, under the assumption of independence over time of dividends and illiquidity costs. However, several previous empirical studies showed that illiquidity is persistent (e.g Chordia, Roll, Subrahmanyam 2000; Huberman, Halka 2001; Hasbrouck, Seppi 2001; Amihud 2002; Pastor, Stambaugh 2003). Hence, the unconditional version of the model (7) can be derived by assuming of a constant risk premium[4] and it may be written as follows:

$$E(R_{i,t}^{G}-R_{F,t}^{G})=E(k_{i,t})+\lambda(\beta_{1,i}+\beta_{2,i}-\beta_{3,i}-\beta_{4,i}) \tag{8}$$

$$\beta_{1,i}=\frac{Cov\left(R_{i,t}^{G},R_{M,t}^{G}-E_{t-1}(R_{M,t}^{G})\right)}{Var(\cdot)} \tag{9}$$

$$\beta_{2,i}=\frac{Cov\left(k_{i,t}-E_{t-1}(k_{i,t}),k_{M,t}-E_{t-1}(k_{M,t})\right)}{Var(\cdot)} \tag{10}$$

$$\beta_{3,i}=\frac{Cov\left(R_{i,t}^{G},k_{M,t}-E_{t-1}(k_{M,t})\right)}{Var(\cdot)} \tag{11}$$

$$\beta_{4,i}=\frac{Cov\left(k_{i,t}-E_{t-1}(k_{i,t}),R_{M,t}^{G}-E_{t-1}(R_{M,t}^{G})\right)}{Var(\cdot)} \tag{12}$$

where $Var_t(\cdot)=Var_t\left[\left(R_{M,t}^{G}-E_{t-1}(R_{M,t}^{G})\right)-\left(k_{M,t}-E_{t-1}(k_{M,t})\right)\right]$ is the variance in the terms from (9) to (12), and $\lambda=E(\lambda_t)=E(R_{M,t}^{G}-k_{M,t}-R_{F,t}^{G})$ is a constant risk premium.

To accommodate the model constraint that the risk premium λ is a constant, Acharya and Pedersen (2005, p. 392) define the "net beta" coefficient of $\beta_{net,P}$ a portfolio P:

$$\beta_{net,P}:=\beta_{1,P}+\beta_{2,P}-\beta_{3,P}-\beta_{4,P} \tag{13}$$

where $\beta_{1,P}$, $\beta_{2,P}$, $\beta_{3,P}$ and $\beta_{4,P}$ are the beta coefficients, which are calculated based on sample covariances and are given by equations from (9) to (12).

Therefore, the LCAPM (8) can be rewritten and becomes:

$$E(R_{P,t}^{G}-R_{F,t}^{G})=\alpha+\kappa\cdot E(k_{P,t})+\lambda\cdot\beta_{net,P} \tag{14}$$

To distinguish the pricing effect of the illiquidity risks from that of the market risk, we can specify the "illiquidity net beta" coefficient $\beta_{5,P}$ of a portfolio P, given by:

$$\beta_{5,P}=\beta_{2,P}-\beta_{3,P}-\beta_{4,P} \tag{15}$$

[4] The LCAPM assumes constant absolute risk aversion A^n of an investor n, but with constant risk aversion the risk premium is approximately constant (Acharya, Pedersen 2005, p. 384).

Finally, the LCAPM (14) can be rewritten using (9) and (15), and becomes:

$$E\left(R_{P,t}^{G}-R_{F,t}^{G}\right)=\alpha+\kappa\cdot E\left(k_{P,t}\right)+\lambda_{1}\cdot\beta_{1,P}+\lambda_{5}\cdot\beta_{5,P} \tag{16}$$

The most general is the following version of the LCAPM:

$$E\left(R_{P,t}^{G}-R_{F,t}^{G}\right)=\alpha+\kappa\cdot E\left(k_{P,t}\right)+\lambda_{1}\cdot\beta_{1,P}+\lambda_{2}\cdot\beta_{2,P}+\lambda_{3}\cdot\beta_{3,P}+\lambda_{4}\cdot\beta_{4,P} \tag{17}$$

In empirical tests of the asset pricing models (14), (16) and (17) the null hypothesis are: $\alpha=0;\ \kappa=0;\ \lambda=0;\ \lambda_i=0,\ i=1,\ldots,5$. The alternative hypothesis are: nonzero intercepts, nonzero premiums for market risk and illiquidity risk, and a positive premium for the illiquidity term $E(k_p)$, i.e. $\alpha\neq 0;\ \kappa>0;\ \lambda\neq 0;\ \lambda_i\neq 0,\ i=1,\ldots,5$, respectively, based on the paper Lee (2011, p. 139). It is pertinent to note that Acharya and Pedersen (2005, p. 393) calibrated κ as the average monthly turnover across all stocks in the sample and it was equal to 0.034. The $E(k_{P,t})$ term in equations (14), (16), and (17) was scaled by κ to adjust for the differences between estimation periods and holding periods. Likewise, we calibrated κ in the whole sample of the WSE-listed stocks used in the study and we got the average monthly turnover κ = 0.002 ≈ 0. Therefore, the empirical results of the LCAPM estimation (Table 5) were almost the same.[5]

Acharya and Pedersen (2005, p. 385) propose a five-step procedure to estimate the LCAPM. As the Polish medium-size emerging stock market is quite different from the U.S. developed market, we modify this algorithm slightly in order to adapt it to the Warsaw Stock Exchange limitations. Details will be presented in the next section.

4. Data description and research procedure

We utilized our own database containing data for the WSE-listed stocks for the period from 2 January 2007 to 28 December 2012. In forming the database, we included securities only if they existed on the WSE for the whole sample period, from 29 December 2006. Finally, the 174 WSE-listed companies were entered into the database. The stock prices and trading volumes were obtained from http://www.gpwinfostrefa.pl. Using the raw data we calculated daily and monthly logarithmic returns, which are appropriate for the asset pricing analysis. Monthly logarithmic returns on the main index (WIG[6]) of the WSE companies were used as the returns on the market portfolio. The monthly average of returns on 52-week Treasury bills were used as returns on a riskless asset. As mentioned in the previous section, we propose a modified version of the Acharya's and Pedersen's (2005) procedure to estimate the LCAPM in the Warsaw Stock Exchange. The procedure consists of seven steps. Empirical results arising from the proposed seven-stage procedure will be presented and discussed in the next section.

In the first step of the procedure, we detect, e.g. based on the DF-GLS test (Elliott, Rothenberg, Stock 1996) or ADF test (Dickey, Fuller 1981) that the analyzed monthly logarithmic return series are

[5] It is important to note that Acharya and Pedersen (2005, p. 394) stress, that the estimation of the LCAPM with κ as a free parameter result in only modest changes in κ and λ.

[6] WIG is a total return index. It accounts for both prices of underlying shares, dividend and subscription rights' income (source: http://www.gpw.pl/indeksy_en).

stationary. The critical values of the DF-GLS or ADF τ-statistic (for the number of data points $T = 50$ or $T = 100$) are presented in Elliott, Rothenberg and Stock (1996), Cook and Manning (2004), MacKinnon (2010). Using monthly data, we utilize maximum lag equal to twelve and then test down to include enough lags so that the last one is statistically significant (Adkins 2013).

In the second step, we estimate the relative illiquidity cost $k_{i,t}$ of each asset i, in each month t of the sample, using the scaled Amihud's measure of illiquidity (18):

$$SILLIQ_t^i = P_{t-1}^M \cdot \frac{1}{D_t^i} \sum_{d=1}^{D_t^i} \frac{\left|R_{t,d}^i\right|}{V_{t,d}^i} \tag{18}$$

In eq. (18) the scaling factor is equal to P_{t-1}^M = WIG at the end of month $(t-1)$/WIG at the end of the base month, while WIG is the main index of the WSE, the base month is December 2006, and notation is as in eq. (1). Similar scaled illiquidity measures were employed by Acharya and Pedersen (2005), Chen (2005), Korajczyk and Sadka (2008).

In the third step, we create three sets of decile test portfolios formed by sorting stocks on the basis of three criteria: the market value *MV*, the Amihud's illiquidity measure, and the standard deviation of the Amihud's illiquidity measure. Acharya and Pedersen (2005) advocated four sets of test portfolios, but we abandoned forming portfolios based on the BV/MV ratio as we found some firms with a negative book-to-market value in our database. We sort all firms according to the annual values of the chosen criteria at the end of each year, beginning in December 2007. As the total number of stocks in the database is 174, we form three sets of decile, equally weighted portfolios following (e.g. Miralles Marcelo, Miralles Quirós 2006; Lee 2011). We compute monthly logarithmic returns and monthly relative illiquidity costs for each portfolio, as well as for the market portfolio in each month. The monthly relative illiquidity cost of an equally weighted portfolio is calculated as an average of the scaled Amihud's measures of illiquidity (18) for stocks included in the portfolio. The postformation returns on these portfolios during the next 12 months are linked across years to form a single return series for each decile portfolio.

The fourth step of the procedure consists of estimating the innovations in illiquidity $k_{P,t} - E_{t-1}(k_{P,t})$ for the market portfolio *M* as well as for each test portfolio *P*, in month *t*. The innovations in illiquidity are necessary for the estimation of the beta coefficients $\beta_{1,P}$, $\beta_{2,P}$, $\beta_{3,P}$, $\beta_{4,P}$, given by equations from (9) to (12). As mentioned in Section 3, illiquidity is persistent. Therefore, Acharya and Pedersen (2005) employ the AR(2) model to calculate the portfolio's illiquidity innovations as follows:

$$SILLIQ_t^P = \alpha_0 + \alpha_1 \cdot SILLIQ_{t-1}^P + \alpha_2 \cdot SILLIQ_{t-2}^P + u_t \tag{19}$$

where $SILLIQ_t^P$ is given by eq. (18) and the innovation in illiquidity for a portfolio *P* in month *t* is defined as $k_{P,t} - E_{t-1}(k_{P,t}) := u_t$.

Likewise, we get the AR(2) model for the market portfolio *M*. Similar approach is advocated in the papers: Pastor, Stambaugh (2003); Korajczyk, Sadka (2006); Sadka (2006); Lee (2011). For comparison, we propose to compute the innovations in illiquidity based on the AR(1) and to choose the best version of the AR(2) or AR(1) models, e.g. based on the Schwarz (SC) information criterion (Tsay 2010). Lower value of the SC measure indicates the preferred model.

In the fifth step of the procedure, we compute the innovations in (gross) returns $R^G_{M,t} - E_{t-1}(R^G_{M,t})$ of the market portfolio *M*. The innovations in (gross) returns are necessary for the estimation of the beta coefficients given by equations from (9) to (12). Acharya and Pedersen (2005) use the AR(2) model to calculate the innovations in returns of the market portfolio, but we advocate to test (e.g. using the SC criterion) whether the AR(1) would be sufficient.

In the sixth step, we calculate and interpret the beta coefficients $\beta_{1,P}$, $\beta_{2,P}$, $\beta_{3,P}$, $\beta_{4,P}$, given by equations from (9) to (12), as well as the parameters $\beta_{net,\,P}$ (13) and $\beta_{5,\,P}$ (15), which are used in the asset pricing models (14), (16) or (17), respectively.[7]

In the last step of the procedure, we estimate the LCAPM. The main goal is to test the hypothesis that illiquidity risk is priced independently of market risk in the Polish medium-size emerging stock market. One of the widely used methodologies is the two-pass regression approach, developed by Fama and MacBeth (1973) and Black, Jensen and Scholes (1972). The Fama-MacBeth procedure (1973) is two--stage and it does have several problems. The regressions are conducted using betas estimated from the time series data, which introduces an errors-in-variables complication. The errors-in-variables problem can be minimized e.g. by grouping the stocks into portfolios and increasing the precision of the beta estimates (Campbell, Lo, MacKinlay 1997). The Fama-MacBeth two-pass methodology (1973) assumes that returns are independent and identically distributed over time. If returns exhibit heteroskedasticity conditional on the factors of serial correlation, the standard errors of the parameter estimates might not be correct, even asymptotically (Cochrane 2005a; Shanken, Zhou 2007). To deal with this problem we apply the robust HAC method[8] by Newey and West (1987) to running cross-sectional regressions of the models (14), (16) or (17). As our sample size is quite small (five years), we advocate to calculate the first-stage betas, given by equations from (9) to (12), based on the whole sample period following Black, Jensen and Scholes (1972).

Additionally, we have used the VIF test to detect for the problem of multicollinearity of betas (Acharya, Pedersen 2005; Lee 2011). The major undesirable consequence of multicollinearity is that the variances of the OLS estimates of the parameters of the collinear variables are quite large. The inverse of the correlation matrix is used in detecting multicollinearity. The diagonal elements of this matrix are called variance inflation factors (VIF). As a rule of thumb, a VIF > 10 indicates harmful collinearity (Maddala 2001).

5. Empirical results and discussion

According to the seven-stage procedure[9] of testing the LCAPM in the Warsaw Stock Exchange, first we detect stationarity of the analyzed monthly logarithmic return series for stocks included in the database. We get that the unit-root hypothesis can be rejected in the case of 150 out of 174 series, while the remaining 24 series are stationary in first differences. Second, we estimate the relative illiquidity cost of each asset, eq. (3), in each month of the sample, using the scaled Amihud's measure of illiquidity,

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[7] In order to reduce the effects of outliers in each series of monthly data, we utilize the procedure presented in Korajczyk and Sadka (2008, p. 49).

[8] HAC – heteroskedasticity and autocorrelation consistent covariance method (Newey, West 1987).

[9] Due to the space restriction we directly report only major results of the seven-stage procedure of testing the LCAPM in the Warsaw Stock Exchange (in Tables 1–5) but details are available upon request.

eq. (18). Third, we create three sets of decile equally weighted test portfolios formed by sorting stocks on the basis of three criteria: the market value MV, the Amihud's illiquidity measure, and the standard deviation of the Amihud's illiquidity measure. We compute monthly logarithmic returns and monthly logarithmic relative illiquidity costs for each portfolio. Table 1 present summarized statistics for the monthly logarithmic returns for three sets of decile equally weighted test portfolios, as well as the statistic testing for normality, in the period from 2 January 2008 to 28 December 2012. Due to the space restriction, we do not present summarized statistics for the monthly relative illiquidity cost series, but details are available upon request.

The measures for skewness and excess kurtosis reported in Table 1 show that the majority of the return series are negatively skewed and leptokurtic with respect to the normal distribution. The Doornik-Hansen test (2008) rejects normality of the monthly logarithmic return series (at the 1% level of significance) in the case of:

1) three out of ten portfolios in the P_{MV} set;

2) three out of ten portfolios in the P_A set;

3) two out of ten portfolios in the P_{VA} set.

In the fourth stage, we calculate innovations in illiquidity for the market portfolio as well as for each test portfolio, in each month. Table 2 summarizes how we choose the appropriate model for calculating innovations in illiquidity for three sets of decile portfolios, based on the data from 2 January 2008 to 28 December 2012. Moreover, as for the market portfolio, the process AR(1) is better fitted compared to the AR(2).

The fifth step of the procedure assumes computing innovations in monthly logarithmic returns of the market portfolio. We compare the results of the estimation based on the AR(1) or AR(2) models and choose the AR(1) process as better fitted (utilizing the Schwarz information criterion).

In the sixth stage we calculate and interpret the beta coefficients. As mentioned in the previous section, we compute the first-stage betas, given by equations from (9) to (12), based on the whole sample period (containing 60 months) following Black, Jensen and Scholes (1972). Table 3 presents the beta coefficients $\beta_{1,P}$, $\beta_{2,P}$, $\beta_{3,P}$, $\beta_{4,P}$ (9)–(12), as well as the parameters $\beta_{net,\,P}$ (13) and $\beta_{5,P}$ (15), estimated for three sets of test portfolios, based on the monthly data from 2 January 2008 to 28 December 2012.

Several results in Table 3 are worth special notice. In Acharya and Pedersen (2005) vein, the beta coefficients (9)–(12) can be interpreted as follows:

1) $\beta_{1,P}$ (9) is the systematic risk measure of portfolio P to changes in the market returns;

2) $\beta_{2,P}$ (10) is the sensitivity measure of illiquidity on portfolio P to changes in the market illiquidity;

3) $\beta_{3,P}$ (11) is the sensitivity measure of the return on portfolio P to changes in the market illiquidity;

4) $\beta_{4,P}$ (12) is the sensitivity measure of illiquidity on portfolio P to changes in the market returns.

As noted in Section 2, the estimates $\beta_{3,P}$ (11) and $\beta_{4,P}$ (12) are expected to be negative. It is worthwhile to note that the results presented in Table 3 are rather typical regarding the sign of the parameters (except for a few results), however, the values of the parameters $\beta_{2,P}$, $\beta_{3,P}$ and $\beta_{4,P}$ are not significantly different from zero. Similarly, the "illiquidity net beta" coefficient $\beta_{5,P}$ (15), which helps to distinguish the pricing effect of the illiquidity risks from that of the market risk, is not significantly different from zero. Hence, the "net beta" coefficient $\beta_{net,P}$ (13) is almost equal to $\beta_{1,P}$ (with accuracy to three decimal places) for all portfolios. Generally speaking, based on the empirical results for beta coefficients one may conclude that the sensitivity measures of various sources of illiquidity are negligible in the Polish emerging stock market.[10]

The cross-sectional regression of the LCAPM is the last step of the procedure. The collinearity of measures of illiquidity risk is confirmed by considering the correlation among the betas, reported in Table 4. The correlations are computed utilizing monthly data in the whole sample period. Overall, besides the evidence of strong collinearity for the variables $\beta_{2,P}$ and $\beta_{3,P}$, we state high values of the VIF measure in the P_{VA} set of test portfolios for the following variables: $E(k_p)$ (VIF = 15.54) and $\beta_{5,P}$ (VIF = 11.47).[11]

Table 5 reports the cross-sectional estimation results of the LCAPM based on the robust HAC method.[12] As the evidence of strong collinearity has been found for the variables $\beta_{2,P}$ and $\beta_{3,P}$, we have problems with estimating of the full version of the asset pricing models in the case of all sets of test portfolios. Therefore, we do not obtain a full version of the model (17), see Table 5.[13]

Table 5 provides evidence that empirical results of the LCAPM allow to reject the hypothesis that illiquidity risk is priced independently of market risk in the Warsaw Stock Exchange. However, it is not surprising if we take into account the results presented in Table 3. The results in Table 5 indicate that only the $\beta_{1,P}$ variable exhibits a significant pattern and only in the case of the P_{MV} set of test portfolios. The $\beta_{5,P}$ variable, which incorporates all sources of illiquidity risk, is not statistically significant. Essentially, there is no doubt as for the intercept because the models imply that the intercept is zero in all cases. Furthermore, the illiquidity term $E(k_p)$ is not statistically significant either. The values of the determination coefficient are low or very low.

On the other hand, the results of the Breusch-Pagan test (1979) for homoskedasticity lend support to the use of the HAC method. Moreover, based on the results of the *F* statistic we have no reason to reject the null hypothesis stating that coefficients are jointly equal to zero in all cases (at 1% significance level).

As a matter of fact, the empirical results of the LCAPM test in the Polish emerging stock market, presented in this paper, are not in accord with the results obtained by Acharya and Pedersen (2005) in the case of the U.S. developed stock market. The authors of the model analyze the period from July 1962 until December 1999, and they find that generally illiquidity risk explains about 1.1% of cross-sectional returns and about 80% of this effect is due to the liquidity sensitivity to the market return. Undoubtedly, it is rather questionable to directly compare these two markets (i.e. in Warsaw and New York) as the U.S. market is arguably the most liquid market in the world. Therefore we avoid such comparisons, but we could try to discuss why the results are different. It has been reported in the literature that, although the problem of illiquidity is present in the Polish medium-size emerging stock market, and it concerns all groups of companies (i.e. small, medium and big firms listed in the WSE), the illiquidity risk is less relevant for asset pricing in the Polish market, e.g. Lischewski, Voronkova (2012). Paradoxically, an equally high level of illiquidity for the majority of companies in the WSE might be the main cause of such results. Essentially, investors require an illiquidity risk premium for assets that

[10] The classical CAPM were estimated for all test portfolios. The results confirmed that the market risk was priced in all cases. The $\beta_{1,P}$ coefficients were statistically significant and even almost equal to those in Table 3, respectively. Details are available upon request.

[11] To avoid a strong collinearity, the variables were orthogonalized but the estimation results were less than satisfactory.

[12] Based on the OLS and heteroskedasticity robust estimators, we obtain the same conclusions concerning the estimated parameters.

[13] All possible variants of the model (17) were estimated, but they did not yield satisfactory results. Therefore, the estimation results of these models are not presented in Table 5.

react strongly to changes in market-wide illiquidity, but in the WSE this reaction is similar for most assets. In other words, in the Warsaw Stock Exchange equity returns do not include an illiquidity risk premium, although various frictions in trading processes are present in the WSE, e.g. Olbryś (2014). It probably might change with the Polish stock market development. Notice here that as of 15 April 2013, the WSE operates the new trading system UTP (Universal Trading Platform), which will create conditions for significant growth in liquidity resulting in potential reduction of trading costs. The UTP proposes new functionalities and services, e.g. new order type, new order validity types, Market Maker System, Bulk/Mass Quotes, High Performance Access, and so on http://www.gpw.pl/trading_system_utp).

6. Conclusions

This paper explicitly tests the research hypothesis that illiquidity risk is not priced in the Polish medium-size emerging stock market. To address this issue, we employ the LCAPM – the equilibrium asset pricing model which includes price impact costs (Acharya, Pedersen 2005). The main advantage of the LCAPM is that it incorporates the trading cost as a random variable into asset pricing, i.e. the trading cost-free stock price is replaced with the price that is adjusted by the stochastic trading cost. Following Acharya and Pedersen (2005) we utilize the illiquidity measure of Amihud (2002) to show how asset prices are affected by illiquidity risk and commonality in liquidity. The LCAPM takes into consideration various sources of illiquidity risk. In contrast to previous studies for the U.S. developed stock market (e.g. Acharya, Pedersen 2005; Pastor, Stambaugh 2003; Avramov, Chordia 2006; Liu 2006), our empirical results indicate no reason to reject the hypothesis that illiquidity risk is not priced in the Warsaw Stock Exchange. These results are rather consistent with the Lischewski and Voronkova (2012) results, although they employed different asset pricing models. Asset pricing with illiquidity risk is an important topic because of many practical implications. From an investor's point of view, the main question is whether less liquid stocks have higher average returns than expected. In other words, investors want to recognize whether they have to take illiquidity risk into consideration in their financial decisions.

Given the uncertainty surrounding illiquidity estimation, a possible direction for further investigation would be to utilize alternative illiquidity measures, which are advocated in the literature to provide empirical research in illiquidity effects in emerging markets. Moreover, the robustness analysis of the LCAPM with respect to various data frequencies could be provided. In light of the recently growing literature on asset pricing, another interesting topic would be to investigate how commonality in liquidity plays a crucial role in non-classical liquidity-adjusted asset pricing. To the best of the author's knowledge, no research concerning commonality in liquidity has been so far undertaken for the Polish stock market.

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Appendix

Table 1

Summarized statistics for the monthly logarithmic returns for three sets of decile test portfolios

Portfolio	Number of stocks	Mean	Standard deviation	Median	Skewness	Excess kurtosis	Doornik--Hansen test
The P_{MV} set; stocks sorted by the market value MV							
P_{MV}^{1}	18	-0.009	0.075	-0.002	-0.69	2.76	14.99 [0.0]
P_{MV}^{2}	17	-0.017	0.085	-0.010	-1.41	5.37	19.14 [0.0]
P_{MV}^{3}	17	-0.017	0.079	-0.021	0.12	1.59	10.22 [0.006]
P_{MV}^{4}	18	-0.017	0.082	-0.010	-0.43	0.94	4.68 [0.096]
P_{MV}^{5}	17	-0.016	0.093	-0.017	0.79	1.17	6.35 [0.04]
P_{MV}^{6}	17	-0.028	0.093	-0.022	-0.13	0.44	2.25 [0.33]
P_{MV}^{7}	18	-0.030	0.085	-0.020	-0.08	0.70	3.68 [0.16]
P_{MV}^{8}	17	-0.011	0.083	-0.012	0.45	1.16	5.83 [0.054]
P_{MV}^{9}	17	-0.021	0.083	-0.015	0.06	0.09	0.77 [0.68]
P_{MV}^{10}	18	-0.011	0.095	-0.019	0.30	1.26	7.05 [0.03]
Mean	–	-0.018	0.085	-0.015	–	–	–
The P_A set; stocks sorted by the Amihud's measure of illiquidity (1)							
P_{A}^{1}	18	-0.018	0.077	-0.022	0.07	0.23	1.26 [0.53]
P_{A}^{2}	17	-0.016	0.087	-0.017	-0.07	2.08	14.77 [0.0]
P_{A}^{3}	17	-0.021	0.078	-0.015	-0.14	0.47	2.37 [0.31]
P_{A}^{4}	18	-0.012	0.073	-0.016	0.08	1.82	12.35 [0.002]
P_{A}^{5}	17	-0.025	0.080	-0.020	0.09	-0.31	0.12 [0.94]
P_{A}^{6}	17	-0.011	0.084	-0.013	-0.24	2.70	19.96 [0.0]
P_{A}^{7}	18	-0.018	0.090	-0.016	-0.06	0.74	3.95 [0.14]
P_{A}^{8}	17	-0.025	0.096	-0.026	-0.21	0.79	4.17 [0.12]
P_{A}^{9}	17	-0.017	0.091	-0.009	-0.16	0.82	4.37 [0.11]
P_{A}^{10}	18	-0.015	0.085	-0.013	-0.49	1.38	6.99 [0.03]
Mean	–	-0.018	0.084	-0.017	–	–	–

The P_{VA} set; stocks sorted by the standard deviation of the Amihud's measure of illiquidity (1)							
P^1_{VA}	18	-0.024	0.073	-0.025	0.04	0.32	1.64 [0.44]
P^2_{VA}	17	-0.018	0.085	-0.007	-0.57	1.75	8.88 [0.01]
P^3_{VA}	17	-0.010	0.092	-0.009	0.70	2.20	10.76 [0.005]
P^4_{VA}	18	-0.013	0.077	-0.019	-0.13	1.19	7.02 [0.03]
P^5_{VA}	17	-0.021	0.072	-0.021	-0.15	-0.31	0.28 [0.87]
P^6_{VA}	17	-0.017	0.081	-0.021	0.03	2.27	16.63 [0.0]
P^7_{VA}	18	-0.021	0.091	-0.017	-0.25	1.36	7.94 [0.02]
P^8_{VA}	17	-0.021	0.093	-0.020	-0.23	0.93	4.98 [0.08]
P^9_{VA}	17	-0.017	0.093	-0.014	-0.04	0.84	4.59 [0.10]
P^{10}_{VA}	18	-0.016	0.083	-0.008	-0.64	1.57	7.48 [0.024]
Mean	–	-0.018	0.084	-0.016	–	–	–

Notes:
The table is based on all observations in the sample during the period 2 January 2008 – 28 December 2012.
The test statistic for skewness and excess kurtosis is the conventional *t*-statistic. The Doornik-Hansen test (2008) has a χ^2 distribution if the null hypothesis of normality is true. The numbers in brackets are *p*-values.
All calculations in this article were done with the Gretl 1.9.11 program.
Notation:

$P_{MV} = \{P^i_{MV}, i = 1,2,\ldots,10\}$ – the set of decile equally weighted test portfolios formed by sorting stocks with the market value *MV*;

$P_A = \{P^i_A, i = 1,2,\ldots,10\}$ – the set of decile equally weighted test portfolios formed by sorting stocks with the Amihud's (2002) illiquidity measure;

$P_{VA} = \{P^i_{VA}, i = 1,2,\ldots,10\}$ – the set of decile equally weighted test portfolios formed by sorting stocks with the standard deviation of the Amihud's (2002) illiquidity measure.

In each of the three portfolio groups $i = 1$ is the portfolio built from assets with the highest value of the respective measure, and $i = 10$ is the portfolio built from assets with the lowest value of that measure.

Table 2

Choosing the best model for calculating innovations in illiquidity for three sets of decile test portfolios

P_{MV}	P^{1}_{MV}	P^{2}_{MV}	P^{3}_{MV}	P^{4}_{MV}	P^{5}_{MV}	P^{6}_{MV}	P^{7}_{MV}	P^{8}_{MV}	P^{9}_{MV}	P^{10}_{MV}
AR(1)		+		+		+	+		+	
AR(2)	+		+		+			+		+
P_{A}	P^{1}_{A}	P^{2}_{A}	P^{3}_{A}	P^{4}_{A}	P^{5}_{A}	P^{6}_{A}	P^{7}_{A}	P^{8}_{A}	P^{9}_{A}	P^{10}_{A}
AR(1)	+	+	+	+	+	+	+	+	+	
AR(2)										+
P_{VA}	P^{1}_{VA}	P^{2}_{VA}	P^{3}_{VA}	P^{4}_{VA}	P^{5}_{VA}	P^{6}_{VA}	P^{7}_{VA}	P^{8}_{VA}	P^{9}_{VA}	P^{10}_{VA}
AR(1)		+	+		+	+	+	+	+	
AR(2)	+			+						+

Notes*:*

The table is based on monthly data for the whole sample in the period 2 January 2008 – 28 December 2012. It presents our choice of the best model for calculating monthly innovations in illiquidity for three sets of decile test portfolios. The AR(1) is a process of order one, while the AR(2) is a process of order two given by (19). The best version of the AR(1) or AR(2) models is chosen based on the Schwarz information criterion.

Remaining notation like in Table 1.

Table 3

The beta coefficients $\beta_{1,P}$, $\beta_{2,P}$, $\beta_{3,P}$, $\beta_{4,P}$ (9)–(12) and the $\beta_{net,P}$ (13) and $\beta_{5,P}$ (15), estimated for three sets of test portfolios

P_{MV}	P^1_{MV}	P^2_{MV}	P^3_{MV}	P^4_{MV}	P^5_{MV}	P^6_{MV}	P^7_{MV}	P^8_{MV}	P^9_{MV}	P^{10}_{MV}
$\beta_{1,P}$	0.986	0.990	0.892	0.973	1.005	1.080	0.953	0.918	0.841	0.873
$\beta_{2,P}$	$9\cdot 10^{-19}$	$5\cdot 10^{-18}$	$2\cdot 10^{-16}$	$2\cdot 10^{-16}$	$2\cdot 10^{-16}$	$-4\cdot 10^{-17}$	$6\cdot 10^{-16}$	$5\cdot 10^{-16}$	$9\cdot 10^{-16}$	$4\cdot 10^{-16}$
$\beta_{3,P}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$
$\beta_{4,P}$	$-2\cdot 10^{-7}$	$-2\cdot 10^{-7}$	$-2\cdot 10^{-6}$	$-4\cdot 10^{-7}$	$4\cdot 10^{-6}$	$-1\cdot 10^{-5}$	$-1\cdot 10^{-5}$	$4\cdot 10^{-6}$	$-3\cdot 10^{-5}$	$-4\cdot 10^{-5}$
$\beta_{net,P}$	0.986	0.990	0.892	0.973	1.005	1.080	0.953	0.918	0.841	0.873
$\beta_{5,P}$	$1.5\cdot 10^{-7}$	$1.8\cdot 10^{-7}$	$2.3\cdot 10\text{-}6$	$3.8\cdot 10^{-7}$	$-4\cdot 10^{-6}$	$1.4\cdot 10^{-5}$	$1.2\cdot 10^{-5}$	$-4.2\cdot 10^{-6}$	$3.3\cdot 10^{-5}$	$4\cdot 10^{-5}$
P_A	P^1_A	P^2_A	P^3_A	P^4_A	P^5_A	P^6_A	P^7_A	P^8_A	P^9_A	P^{10}_A
$\beta_{1,P}$	0.830	0.865	0.866	0.796	0.819	0.956	1.060	1.111	1.094	1.111
$\beta_{2,P}$	$1\cdot 10^{-15}$	$8\cdot 10^{-16}$	$-9\cdot 10^{-17}$	$2\cdot 10^{-16}$	$1\cdot 10^{-16}$	$3\cdot 10^{-17}$	$1\cdot 10^{-16}$	$3\cdot 10^{-17}$	$1\cdot 10^{-18}$	$3\cdot 10^{-19}$
$\beta_{3,P}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$
$\beta_{4,P}$	$-3\cdot 10^{-5}$	$-5\cdot 10^{-5}$	$-1\cdot 10^{-6}$	$9\cdot 10^{-6}$	$-8\cdot 10^{-6}$	$8\cdot 10^{-7}$	$-4\cdot 10^{-7}$	$-1\cdot 10^{-6}$	$-6\cdot 10^{-8}$	$-7\cdot 10^{-9}$
$\beta_{net,P}$	0.830	0.865	0.866	0.796	0.819	0.956	1.060	1.111	1.094	1.111
$\beta_{5,P}$	$3.4\cdot 10^{-5}$	$4.7\cdot 10^{-5}$	$1.4\cdot 10^{-6}$	$-9.4\cdot 10^{-6}$	$7.7\cdot 10^{-6}$	$-7.5\cdot 10^{-7}$	$4.4\cdot 10^{-7}$	$1.2\cdot 10^{-6}$	$6.5\cdot 10^{-8}$	$6.6\cdot 10^{-9}$
P_{VA}	P^1_{VA}	P^2_{VA}	P^3_{VA}	P^4_{VA}	P^5_{VA}	P^6_{VA}	P^7_{VA}	P^8_{VA}	P^9_{VA}	P^{10}_{VA}
$\beta_{1,P}$	0.797	0.891	0.929	0.896	0.755	0.871	1.083	1.045	1.128	1.086
$\beta_{2,P}$	$1\cdot 10^{-15}$	$1\cdot 10^{-15}$	$-3\cdot 10^{-16}$	$2\cdot 10^{-16}$	$2\cdot 10^{-16}$	$5\cdot 10^{-18}$	$1\cdot 10^{-16}$	$2\cdot 10^{-17}$	$1\cdot 10^{-18}$	$3\cdot 10^{-19}$
$\beta_{3,P}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$	$-2\cdot 10^{-11}$
$\beta_{4,P}$	$-7\cdot 10^{-5}$	$-2\cdot 10^{-5}$	$-8\cdot 10^{-7}$	$2\cdot 10^{-6}$	$7\cdot 10^{-6}$	$-4\cdot 10^{-6}$	$-1\cdot 10^{-7}$	$-9\cdot 10^{-7}$	$-7\cdot 10^{-8}$	$-7\cdot 10^{-9}$
$\beta_{net,P}$	0.797	0.891	0.929	0.896	0.755	0.871	1.083	1.045	1.128	1.086
$\beta_{5,P}$	$6.9\cdot 10^{-5}$	$2.4\cdot 10^{-5}$	$7.9\cdot 10^{-7}$	$-1.6\cdot 10^{-6}$	$-7.3\cdot 10^{-6}$	$4\cdot 10^{-6}$	$-1\cdot 10^{-7}$	$9.2\cdot 10^{-7}$	$7\cdot 10^{-8}$	$6.5\cdot 10^{-9}$

Notes:

The table is based on monthly data for the whole sample in the period 2 January 2008 – 28 December 2012. It presents the beta coefficients $\beta_{1,P}$, $\beta_{2,P}$, $\beta_{3,P}$, $\beta_{4,P}$, $\beta_{net,P}$, and $\beta_{5,P}$ obtained based on equations (9)–(12), (13) or (15), for three sets of decile test portfolios, respectively.

Remaining notation like in Table 1.

Table 4

Beta correlations for three sets of test portfolios

	$\beta_{1,P}$	$\beta_{2,P}$	$\beta_{3,P}$	$\beta_{4,P}$
		$\boldsymbol{P_{MV}}$		
$\beta_{1,P}$	1.000	-0.750	-0.519	0.518
$\beta_{2,P}$		1.000	0.593	-0.553
$\beta_{3,P}$			1.000	-0.089
$\beta_{4,P}$				1.000
		$\boldsymbol{P_{A}}$		
	$\beta_{1,P}$	$\beta_{2,P}$	$\beta_{3,P}$	$\beta_{4,P}$
$\beta_{1,P}$	1.000	-0.496	-0.886	0.360
$\beta_{2,P}$		1.000	0.576	-0.877
$\beta_{3,P}$			1.000	-0.496
$\beta_{4,P}$				1.000
		$\boldsymbol{P_{VA}}$		
	$\beta_{1,P}$	$\beta_{2,P}$	$\beta_{3,P}$	$\beta_{4,P}$
$\beta_{1,P}$	1.000	-0.494	-0.786	0.399
$\beta_{2,P}$		1.000	0.446	-0.915
$\beta_{3,P}$			1.000	-0.476
$\beta_{4,P}$				1.000

Notes:
The table is based on monthly data for the whole sample in the period 2 January 2008 – 28 December 2012. It reports the correlations of $\beta_{1,P}$, $\beta_{2,P}$, $\beta_{3,P}$, and $\beta_{4,P}$ for three sets of decile test portfolios. The critical value of the correlation coefficient is equal to 0.632 (at 5% significance level and $n = 10$).
Notation like in Tables 1–3.

Table 5

The LCAPM cross-sectional regression results for three sets of test portfolios

The P_{MV} set of test portfolios												
Model (14)	intercept	$E(k_P)$	–	–	–	–	–	$\beta_{net,P}$	R^2	χ^2	BP	F
1	0.044 (0.024)	-740.66 (427.0)	–	–	–	–	–	-0.064 (0.023)	0.360	0.020 [0.990]	1.118 [0.572]	4.002 [0.069]
Model (16)	intercept	$E(k_P)$	$\beta_{1,P}$	–	–	–	$\beta_{5,P}$	–	R^2	χ^2	BP	F
1	0.043 (0.026)	-860.7 (932.9)	-0.063 (0.025)	–	–	–	52.70 (368.8)	–	0.363	0.220 [0.896]	1.202 [0.752]	2.254 [0.182]
Model (17)	intercept	$E(k_p)$	$\beta_{1,P}$	$\beta_{2,P}$	$\beta_{3,P}$	$\beta_{4,P}$	–	–	R^2	χ^2	BP	F
1	0.043 (0.026)	-860.7 (932.9)	-0.063 (0.025)	1	–	-52.70 (368.8)	–	–	0.363	0.220 [0.896]	1.202 [0.752]	2.254 [0.182]
The P_A set of test portfolios												
Model (14)	intercept	$E(k_P)$	–	–	–	–	–	$\beta_{net,P}$	R^2	χ^2	BP	F
1	-0.021 (0.022)	49.05 (226.4)	–	–	–	–	–	-0.001 (0.021)	0.015	0.064 [0.968]	1.879 [0.391]	0.144 [0.868]
Model (16)	intercept	$E(k_P)$	$\beta_{1,P}$	–	–	–	$\beta_{5,P}$	–	R^2	χ^2	BP	F
1	-0.025 (0.022)	279.8 (435.9)	0.003 (0.021)	–	–	–	-119.9 (183.9)	–	0.077	0.482 [0.786]	0.563 [0.905]	0.220 [0.879]
Model (17)	intercept	$E(k_P)$	$\beta_{1,P}$	$\beta_{2,P}$	$\beta_{3,P}$	$\beta_{4,P}$	–	–	R^2	χ^2	BP	F
1	-0.025 (0.022)	279.8 (435.9)	0.003 (0.021)	–	–	119.9 (183.9)	–	–	0.077	0.482 [0.786]	0.563 [0.905]	0.220 [0.879]
The P_{VA} set of test portfolios												
Model (14)	intercept	$E(k_P)$	–	–	–	–	–	$\beta_{net,P}$	R^2	χ^2	BP	F
1	-0.019 (0.013)	-185.5 (170.3)	–	–	–	–	–	-0.002 (0.013)	0.129	1.218 [0.544]	0.275 [0.872]	0.987 [0.419]
Model (16)	intercept	$E(k_P)$	$\beta_{1,P}$	–	–	–	$\beta_{5,P}$	–	R^2	χ^2	BP	F
1	-0.038 (0.021)	772.6 (621.3)	0.015 (0.019)	–	–	–	-342.7 (201.9)	–	0.420	0.779 [0.677]	2.218 [0.528]	5.171 [0.042]
Model (17)	intercept	$E(k_P)$	$\beta_{1,P}$	$\beta_{2,P}$	$\beta_{3,P}$	$\beta_{4,P}$	–	–	R^2	χ^2	BP	F
1	-0.038 (0.021)	772.6 (621.3)	0.015 (0.019)	–	–	342.7 (201.9)	–	–	0.420	0.779 [0.677]	2.218 [0.528]	5.171 [0.042]

Notes:

The table reports the LCAPM cross-sectional regression results of the models (14), (16), and (17), based on the robust HAC method (Newey, Wes 1987). The heteroskedastic consistent standard errors are in parentheses below the coefficient estimates. The Doornik-Hansen test (2008) (χ^2) has a chi-squared distribution if the null hypothesis of normality is true. The Breusch-Pagan test (1979) (BP) has a chi-squared distribution if the null hypothesis of homoskedasticity is true. R^2 denotes a determination coefficient. The last column reports the F statistics to verify the null hypothesis that coefficients are jointly equal to zero. The numbers in brackets are p-values. As for the collinearity of the variables $\beta_{2,P}$ and $\beta_{3,P}$ in the case of all sets of test portfolios we do not obtain a full version of the model (17).

Notation like in Tables 1–4.