Options and Market Expectations: Implied Probability Density Functions on the Polish Foreign Exchange Market*

Opcje a oczekiwania rynku: estymacja i wykorzystanie implikowanych funkcji gęstości prawdopodobieństwa na polskim rynku walutowym

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Abstract

An overview of methods used for estimation of option-implied risk-neutral probability density functions (PDFs) is presented in the study, and one of such methods, double lognormal approach, is used for the analysis of the information content of the EUR/PLN currency options on the Polish market. Estimated PDFs have proven to provide superior information concerning future volatility than historical volatility, yet their forecasting power is comparable to that of the Black-Scholes model. There are no strong grounds for using PDFs as a predictor of the future EUR/PLN exchange rate. Low informative content does not directly follow, as PDFs can be used as an indicator of markets conditions. The issues that could be addressed more thoroughly in the future studies concern the assumption of risk neutrality and the impact of the estimation method on the higher moments of the distribution.

Keywords: foreign exchange, probability density functions, option pricing, market expectations

JEL: F31, G13, D84

Streszczenie

W artykule dokonano przeglądu metod estymacji funkcji gęstości prawdopodobieństwa (PDF) instrumentu bazowego na podstawie cen opcji przy założeniu neutralności wobec ryzyka. W analizie rynku EUR/PLN zastosowano metodę dwóch rozkładów logarytmiczno-normalnych. Okazało się, że oszacowane PDF dostarczają więcej informacji o przyszłej zmienności niż zmienność historyczna, jednak ich zawartość informacyjna była bardzo zbliżona do oferowanej przez model Blacka-Scholesa. Brak jest silnych podstaw do użycia kontraktów opcyjnych jako instrumentu prognozującego przyszłe poziomy kursu EUR/PLN. Nie jest to jednak tożsame z niską zawartością informacyjną PDF, które mogą być użyte jako wskaźnik sytuacji na rynku. Elementy, które zasługują na pogłębioną analizę, to założenie o neutralności wobec ryzyka oraz wpływ metody estymacji na wyższe momenty implikowanych rozkładów prawdopodobieństwa.

Słowa kluczowe: kurs walutowy, funkcje gęstości prawdopodobieństwa, wycena opcji, oczekiwania rynku

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1. Introduction

Contrary to many other financial instruments, the price of which reflects all market scenarios, options can be used to "show" probabilities attached by investors to particular events. One can obtain such information through estimation of option implied probability density functions (PDF). While this type of analysis has become increasingly popular in recent years, the number of publications in many areas of this field remains relatively limited. Most of the research has concentrated on developing, testing and comparing characteristics of new estimation techniques, whereas less attention has been paid to the analysis of their information content and forecasting power. This paper seeks to go in the latter direction and investigate the forecasting power of 1-month option contracts on the Polish foreign exchange market – namely for the EUR/PLN currency pair.

In the standard Black-Scholes option pricing model, it is assumed that the distribution of the underlying instrument is of a lognormal type.[1] Yet market prices of option contracts indicate that investors make different assumptions. These discrepancies give grounds to the analysis of options' market quotes in order to estimate market-expected distributions of the underlying instrument, thus providing information on the expected rates of return or probability attached to particular events (realisation of a given currency level, equity price).

It should be stressed that the analysis is conducted under the assumption of risk neutrality. In some situations, such an assumption may prove to be inappropriate, resulting in bias and significant discrepancies between estimated option implied probability and subjective probability as seen by investors. Taking into consideration the unresolved difficulties in capturing agents' preferences under different states of nature, the risk neutrality assumption dominates in works on option implied PDFs.

The paper is structured as follows. In the first part, a theoretical basis for option pricing is presented, together with a list of major anomalies between market practice and the Black-Scholes model. In particular, option prices on the market do not seem to coincide with the assumption of lognormal distribution of the underlying asset. Such observation provides basis for analysis that aims at estimating how this PDF really looks like. The second section explains how option prices can be translated into probabilities attached by market participants to certain events. The third section describes three major groups of methods that are used to estimate implied PDFs. The fourth section is dedicated to a more detailed presentation of the double lognormal approach, applied for the EUR/PLN currency pair. In the last section, the information content of the EUR/PLN option implied PDF is investigated.

[1] A lognormal distribution is a distribution of a variable whose logarithm has a normal distribution.

2. Black-Scholes model

An option[2] gives its buyer the right to buy or sell a given underlying instrument at the expiry date at the previously set price (strike). An option is an asymmetrical instrument as its seller has the obligation to execute a transaction on buyer's demand at the expiry.

Valuation of each derivative requires identification of the stochastic process that governs the evolution of the underlying instrument's price. Initially, Bachelier (1900) in his work on pricing bond option assumed that bond prices evolved according to the arithmetic Brownian motion. Still long horizon of expiry of some bonds allowed for option prices to reach negative values, which distorted valuation. An assumption of the geometric Brownian motion introduced by Samuelson (1965) for equity valuation eliminated this inconveniency. Yet his model required estimation of two important factors: the expected rate of return on equities and the discount rate. As both factors depended on investors' utility functions, such a method of valuation suffered from the necessity of making a strong assumption on the above parameters.

Black and Scholes (1973) developed a completely novel approach to this subject. Their starting point was to analyse the transaction from the point of view of the option seller who wishes to hedge his position. As they have shown, along with an increase in hedging frequency the cost of hedging itself becomes increasingly easier to anticipate. In limiting case, where hedging activity is continuous, its cost is independent from the price of the underlying asset. The single factor that influences this cost is the variance (volatility) of the asset price. Should this variable be known in advance, it would allow for fair option valuation.

In the Black-Scholes (1973) model, it is assumed that the price of the underlying instrument evolves according to the geometric Brownian motion, which means that asset's price can be characterised by a lognormal distribution with a constant variance. It is further assumed that the risk free rate is constant till the maturity of the contract, investors can lend and borrow at this rate and there are no transaction costs. We shall try to touch upon these assumptions later in the text.

Garman and Kohlhagen (1983) have adopted the Black-Scholes (B-S) model to currency options. Taking a similar assumption, they have shown that the prices of call (c) and put (p) options can be given by the following formulas:[3]

[2] We constrain our analysis to the European option which can be executed only at the expiry date, contrary to the American option.

[3] The Garman-Kohlhagen (G-K) model does not account for discrepancies between the stochastic time (between transaction date and expiry date) and swap time (between premium being paid and final settlement of the contract). Stochastic time is linked to implied volatility of the underlying asset and swap time is linked to interest rates. The failure to account for the difference between the two measures results in incorrect option valuation. For further reading, see Stopczyński, Węgleńska (1999). Taking into consideration that possible bias due to this phenomenon is most probably significantly lower than the bias due to quality of the data, we will proceed with the analysis without correcting the equations of the G-K model.

$$c = e^{-r_f \tau} SN(d_1) - e^{-r\tau} XN(d_2) \quad (1)$$

$$p = e^{-r\tau} XN(-d_2) - e^{-r_f \tau} SN(-d_1) \quad (2)$$

where:

$$d_1 = \frac{\ln(S/X) + (r - r_f + \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \quad (3)$$

$$d_2 = d_1 - \sigma\sqrt{\tau} \quad (4)$$

r and r_f – domestic and foreign risk-free rates,

τ – time to expiry (in years),

σ – standard deviation (volatility) of the underlying instrument,

S – spot foreign exchange,

X – strike price,

$N(d)$ – standard normal distribution $N(0, 1)$.

From the above equations one may see that the only unknown variable at the time when option is being priced is the volatility of the underlying instrument.[4] That is why market makers, especially on the foreign exchange market, do not quote price at which they are ready to buy or sell an option, but the (implied) volatility. This volatility can be used to calculate option's price and thus premium to be paid via B-S model.[5] What is important to note is that market participants do not have to "believe" in the B-S assumption to use it, as the model serves as a clear-cut transformation from volatility to prices. In other words, market makers quoting volatility obtain unequivocal information on prices they are to be paid for selling or buying a given option. Moreover, it should be noted that option-implied volatility reflects price offsetting demand with supply for options; it does not necessarily have to be equal to the expected volatility.

2.1. Anomalies in option prices – smiles and smirks

The Black-Scholes model (and its extensions) is the fundamental method for option pricing on the market. Still, market participants do alter some of the assumptions of the model that result as "anomalies" – discrepancies between B-S model implications and market quotes.

One trait of many assets' prices, including foreign exchange, is that the process governing their evolution is not of continuous nature[6] (Micu 2005). Foreign exchange dynamics following the publication of important data may provide an example of such discontinuous adjustments. The speed and scale of price changes can be so big that they make it virtually impossible for the option seller to hedge his exposure on continuous terms. This factor, together with transaction costs, does not allow market makers to hedge their position as effectively as it is assumed in the B-S model. To mitigate these effects, an additional premium must be included in option prices, which translates into higher quotes of implied volatility. Still, alternative valuation models that extend the B-S model to account for the above factors, such as the stochastic volatility model (Hull, White 1987; Heston 1993) or models based on jump-diffusion process (Merton 1976; Bates 1991; 1996a; 1996b), have also failed to mimic option prices on the market (Bates 2000).

The main difference between the B-S model and market practice concerns the shape of the volatility surface. Volatility surface can be generated through combining implied volatilities for different option maturities (term structure) with implied volatility for different strike prices. Information about volatility surface allows for direct valuation of any European-style option and through PDF analysis to estimate probabilities of various market scenarios in many time horizons. The B-S model implies that the volatility is equal across all strike prices and across all time horizons – thus, the volatility surface is completely flat. Even without the assumption of continuous price generating process and non-existing transaction costs, the volatility surface would tend to be higher for all maturities and all strikes in a similar scale. In reality, this does not happen and there are two main groups of anomalies between market-implied volatility surface and that implied by the B-S model. These are:

– volatility smile

– volatility term structure

Volatility term structure takes its name after the fact that implied volatility (market quotes) differs across time horizon, contrary to what the geometric Brownian motion assumption of the B-S model implies. The stylised fact on developed markets is that implied volatility rises with options' maturity, which may stem from its expected increase or from the willingness to pay additional premium for being able to hedge against detrimental price changes at longer horizons (Campa, Chang 1995).

A volatility smile refers to the situation where out-of-the-money (OTM) options exhibit higher implied volatility than at-the-money options.[7] It means that implied volatility increases along with the distance between option's strike price and forward price. A volatility smile implies that investors do not value options assuming that the stochastic process governing the price evolution of the underlying instrument is a geometric Brownian motion. Such a phenomenon

[4] The second variable is the interest rate which does not necessarily have to stay constant till the expiry. Still, market practice is that it is assumed to be equal to the interest rate with the same maturity as the option.

[5] We use B-S abbreviation for currency options valuation model, though it is the G-K model that is applied. This is due to the fact that the G-K model can be treated as an extension of the B-S model to the currency market.

[6] B-S model assumes that this process is continuous; there are no big, sudden price changes (jumps).

[7] Option is called at-the-money (ATM) when the strike price is equal to the current price of the underlying instrument. Call (put) options are called out-of-the-money when the strike price is higher (lower) than the current price of the underlying instrument. For in-the-money options, the situation is symmetrical to the out-of-the-money case.

translates into implied leptokurtic distribution of the underlying asset (under risk-neutrality), so that the probabilities of extreme events are higher than in the lognormal distribution (fat tails effect).

The assumption of the lognormal distribution (implied by the geometric Brownian motion) was rejected in several studies on financial instruments' price behaviour – Hawkins et al. (1996); Sherrick et al. (1996); Jondeau, Rockinger (2000); Navatte, Villa (2000) and Bahra (2001).

The second phenomenon associated with volatility surface is a volatility smirk. A volatility smirk is characteristic for the equity market and for some emerging currency markets. It refers to the situation when OTM put option volatility differs from OTM call option volatility (both options having the same absolute delta values). In the case of the volatility smile, the implied volatility increased in symmetrical way along with the distance between the strike and forward price regardless of the direction (below or above forward). 'Smirk' means that this augment is not symmetrical and happens to be bigger in the space above or below forward price.

With a volatility smirk, the implied probability of a significant price increase is not equal to the probability of a significant price decrease (with the assumption of risk neutrality – more below). A volatility smirk manifests in non-zero prices of risk reversal contracts[8] and implies skew in the expected distribution of the rates of return of the underlying instrument (assuming risk neutrality). For example, the implied volatility of the options that allow to sell one of the core markets currency (EUR, USD) against the emerging market currency below the current spot price (OTM option) tends to be lower than the implied volatility of the mirror call option with the strike price above spot price (also OTM option).[9] Persistent difference between OTM call and put options may imply the so-called *peso problem*, a low-probability sudden decrease in value of the emerging currency (Micu 2005). Such a situation highlights the importance of risk-neutrality assumption. We shall come back to this topic later.

On the stock market, equity put options with strikes below current price exhibit higher implied volatility than call options with strikes above a current price (both with the same delta values). This volatility smirk has become pronounced after the Black Monday on 19 October 1987, when US stock markets plunged over 20% in a single day. This is why Rubinstein (1994) describes the volatility smirk on the equity market as "*crash-o-phobia*". He suggests that investors may be afraid that such an event may re-occur, which leads to an increase in costs of protection against it (higher OTM put option prices). One can think of a currency crisis as of a stock crush happening on the foreign exchange market. A relatively large number of such extreme events materialising recently on emerging markets may, to some extent, explain the presence of a volatility smirk among emerging currencies.

3. Probability density function and Arrow-Debreu security

Due to their substantial information content, option-implied PDFs have become an increasingly popular target of scientific analysis. The prices of financial instruments reflect all (probability weighted) market-possible scenarios, yet in most of the cases it is extremely difficult, if not impossible, to derive probabilities attached to realisation of a particular event. The prime advantage of the option market is that through PDF analysis one can "show" or "see" the implied probability of any event. Such a type of research, which aims at derivation of market scenarios from the option market, can be found in Rubinstein (1994) or Aït-Sahalia, Lo (1995), where the probability of a stock market crash was estimated. On the same grounds, Melick and Thomas (1997) investigate the probability of the outcome of the Gulf War (war-peace) during the conflict in 1991; Leahy and Thomas (1996) conducted research on uncertainty due to the referendum over Quebec's independence in 1995; whereas Campa, Chang and Reider (1997; 1998) investigate consistency between official fluctuation bands in various currency regimes (like ERM) and option-implied market expectations.

The idea behind option implied PDF analysis is as follows. Assuming risk neutrality among investors, the difference between prices of two options having the same maturity but different strikes (agreed price of the underlying instrument in respect to which the transaction is to be settled at maturity) reflects probabilities that market participants attach to particular events materialising in the future.

A special case of an instrument, which has its price dependent on the future state, is an Arrow-Debreu security (Arrow 1964; Debreu 1959). This is a derivative where the buyer receives payoff of "1" if an underlying instrument takes a certain state at maturity and "0" otherwise. Assuming risk neutrality price of an Arrow-Debreu security for a certain state is directly proportional to the probability of this state to materialise.

[8] Risk reversal (RR) on the foreign exchange market consists of two option contracts (long call option and short put option), both having the same absolute delta value. RR offer price is given as a difference between long call option volatility and short put option; bid price is taken as a difference between short call option and long put option. In other words, RR is a difference between volatility on the right side of the volatility smirk and the volatility on the left side of the smirk. In this work, RR is taken as a mean of the two prices. The most popular RR contract is 25D which consists of two options both having delta equal to 0.25. One can think of the delta as a probability for the option to expire in-the-money.

[9] I.e. EUR/PLN OTM put option (strike below the current spot) has lower implied volatility than EUR/PLN OTM call option (strike over the current spot), both having the same absolute delta values.

Under further assumption of complete markets, where instruments for all possible future states are traded, it would be possible to derive the whole probability density functions of any asset (underlying instrument). The information content of such a derivative is substantial.

An Arrow-Debreu security can be replicated via options by combining them into a strategy called *butterfly spread.*[10] Ross (1976) showed that having option prices for all possible future states (strikes) would allow us to recreate the PDF of the underlying instrument.

In reality, there are relatively few states (strikes) for which a liquid option market exists. In most of the cases, these are six price levels of the underlying instrument – according to market convention, these levels correspond to certain levels of option's delta and for strike equal to forward price. Delta can be roughly seen as an expected probability of an option buyer executing the transaction at maturity; its absolute value falls between 0 and 1.[11] In other words, for a call (put) option this is approximately the expected probability that the future price of the underlying instrument at the maturity is above (below) strike. Formally, delta is the first derivative of options' price with respect to underlying instruments' price.

Thus, the number of liquid options across the strike space (underlying instruments' price) is far from dense. Estimation techniques generally tend to optimise some conditions to find the PDF that best fits the data. Final PDF is found by interpolation between these knots and extrapolation outside them. In the next section, we are going to describe methods that are currently used for PDF estimation with a limited amount of options available and one of such methods shall be applied to the EUR/PLN exchange rate.

4. Probability density function estimation methods

Three main groups for PDF estimation can be identified (BIS 1999; Syrdal 2002):

(i) explicit assumption concerning type of stochastic process governing price evolution of the underlying instrument,

(ii) founded on Cox-Ross (1976) equation,

(iii) founded on Breeden-Litzberger (1978) equation.

The second and third group dominates in the literature, their popularity relatively equal with neither of them strictly overwhelming the other.

Ad (i)

In this method, one first makes assumptions concerning characteristics of the stochastic process governing price evolution of the underlying instrument and then uses market option prices to estimate the parameters of the process. Probability density function is thus given as a by-product. This approach has been used by Bates (1991; 1996a; 1996b) and Malz (1996). Still, this is one of the least popular methods, partly due to its relatively small flexibility. Making the assumption concerning the stochastic process of the underlying instrument implies strong restrictions on the type of the PDF. This is due to the fact that a given stochastic process cannot have many types of distribution, it has only one. The advantage of this approach over other methods is that once a stochastic process is identified, one can use this method to replicate options and hedge his exposure. It is worth mentioning that valuation models that assumed a stochastic process different to the B-S (*stochastic volatility models* – Hull, White 1987; Heston 1993; *jump diffusion process* – Merton 1976; Bates 1991) have failed to generate option prices matching these observed on the market (Bates 2000). PDF implied from such methods may thus have serious problems with mirroring investors' expectations. If one aims at deriving market expectations, he should rather focus on methods (ii) and (iii).

Ad (ii)

The idea behind this method is to make some assumptions concerning the type of the distribution (PDF), and not the stochastic process itself. The advantage of such an approach is that while a stochastic process has only one corresponding distribution, a single distribution can be generated by different stochastic processes. This makes method (ii) less restrictive than (i).

Cox and Ross (1976) have shown that assuming risk neutrality option's price can be expressed as an expected value of its future values discounted with risk-free rate. Moreover, the option's expected value depends on the probability density function. Formally:

$$c = e^{-r\tau}\int_X^{\infty} q(S_T,\gamma)(S_T - X)dS_T \qquad (5)$$

$$p = e^{-r\tau}\int_0^{X} q(S_T,\gamma)(X - S_T)dS_T \qquad (6)$$

where $c(p)$ is the price of the call (put) option, X is the strike price, τ is time to maturity, and $q(S_T, \gamma)$ is the PDF of the underlying instrument S_T at the maturity T which is characterised by the vector of the parameters.

Should infinitely dense strike space was available, there would exist only one PDF matching the data. Given the fact that only few market prices are at hand, techniques based on C-R equation generally make some assumptions concerning the characteristics of the risk-neutral PDF $q(S_T, \gamma)$, whose parameters γ (such as mean and standard deviation) are estimated to minimise the difference between theoretical option prices implied by the equations above and market prices.

[10] This combination consists of a sale of two call options with strike X and purchase of two call options with strikes $X + \Delta x$ and $X - \Delta x$. In practice, *butterfly spread* is constructed out of *straddle* and *strangle* strategies – two call options (ATM and OTM) and two put options (ATM and OTM as well).

[11] For some exotic options it can take higher values.

One of the most popular approaches within this method is the so-called *mixture of lognormals*, applied, among others, by Melick and Thomas (1997); Mizrach (1996); Söderlind and Svensson (1997). PDF is generated by a certain number (most commonly two, hence the name *double lognormal*) of independent lognormal distributions. The final shape of the PDF is sufficiently flexible to accommodate such effects like pronounced skewness or fat tails. We shall tackle this approach more in depth in the sections to follow.

Researchers working on the implied PDF have also turned to more general types of distribution that are even less restrictive. These include g and h distribution (Dutta, Babel 2005), second type beta distribution (McDonald, Bookstaber 1991; Bookstaber, McDonald 1987; McDonald 1996; Rebonato 1999), Burr III distribution (Sherick et al. 1996) and Weibull distribution (Savickas 2001).

Works of Madan and Milne (1994) as well as Corrado and Su (1998) provide an example of a different approach, where as a starting point for estimation normal distribution is taken and is further corrected via Hermite polynomial. Rubinstein (1994) starts with a lognormal distribution and accounts for bid-ask spread in option prices. Among many types of distribution, he seeks one that corresponds to the bid-ask restrictions and is most similar to the initial lognormal distribution. Buchen and Kelley (1996) go in a similar direction, yet for optimisation procedure they use maximum entropy instead of least squares.

Ad (iii)

A connection between option prices and PDF was formally provided by Breeden and Litzberger (1978). They have shown[12] that the second derivative of options' price with respect to strike is directly proportional to the underlying instrument's probability density function and under the assumption of risk-neutrality the following relation exists:

$$\frac{\partial^2 c(X,\tau)}{\partial X^2} = e^{-r\tau} q(S_T) \qquad (7)$$

where c is options' price, X corresponds to strike, τ is time to maturity and $q(S_T)$ is the PDF of the underlying instrument S_T. The big advantage of this approach is that it does not make any assumptions concerning underlying instruments' price dynamics. Methods based on the Breeden-Litzberger (B-L) equation estimate parameters of the function that binds call option prices with the strike, so that after differentiating twice, PDF can be obtained.[13] While methods based on the Cox-Ross (C-R) equation were directly selecting (optimising) among the possible PDFs, methods based on the B-L formula estimate the function relating option prices with strikes as a first step and then differentiate this function to obtain PDF. Moreover, in some of the cases, PDF tails must be fitted separately.

Shimko (1993) was among the very first to apply this approach. He first uses the B-S formula to change the *strike price-option price* space into the *strike price-implied volatility* space. Based on market quotes of volatility, a quadratic function relating option prices with volatility using the B-S model is estimated. Once this is achieved, one can express implied volatility as a function of the strike price. Differentiating twice, along the B-L equation, PDF is obtained. Still, the tails of the distribution that lack data must be estimated separately. Shimko (1993) uses the lognormal distribution that is fitted into tails so that the integral on the whole PDF sums up to one. Malz (1997) follows a similar path, yet in place of strikes he uses option's deltas, which allows for a better fit of the PDF. Campa, Cheng and Reider (1998) who also base on the Shimko's approach use splines instead of a quadratic equation. This provides them with greater estimation flexibility, leading to better PDF fit. Bliss and Panigirtzoglou (2000) combine Malz's (1997) approach of taking volatility at certain delta values with spline estimation adopted by Campa, Cheng and Reider (1998).

Aït-Sahalia and Lo (1998) take quite a different path. Instead of estimating the relation between options' price and strike at each point of time separately, as was done by other researchers, they estimate the general form of the function for the whole time series available. To this end, Naradya-Watsan kernel estimator is applied, yet this approach tends to be extremely time consuming.

It must be stressed that all of the above mentioned methods and approaches of PDF estimation implicitly or explicitly assume risk-neutrality among investors. Taking option prices from the risk-averse world to estimate PDF in the risk-neutral model can result in a serious bias. In such a case, an event (state) with small occurrence probability in the world characterised by risk aversion may have substantially higher probability attached to it in the world assuming risk-neutrality. This is due to the fact that risk aversion, or even loss aversion, implies that certain states of nature will have a relatively high value attached to them, not necessarily reflecting true probability of occurrence, but simply people's willingness to pay substantially more for the insurance against them than the actuarial cost. The final price of the instrument will thus depend on two factors – subjective probability for the state to materialise and utility attached to it. While operating in risk-neutral model we shall not be able to distinguish between the two, which may lead to overstatement of certain probabilities. Only recently some researches have started to tackle risk-aversion in PDF estimation

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12 Assuming there are no transaction costs, no restrictions on short sale and investors can lend and borrow at risk-free rate.

13 After single differentiation one can obtain cumulated density function.

(Aït-Sahalia, Lo 2000; Jackwerth 2000; Rosenberg, Engle 2002). Still, the problem of capturing agents' utility across different states of nature remains unresolved and the risk-neutrality assumption remains the dominant approach in PDF estimation studies. Bearing this in mind, one should be careful when drawing conclusions concerning market expectations derived from option implied risk-neutral PDFs.

4.1. Methods comparison

Taking into account the number of approaches towards option implied risk-neutral PDF estimation, one would most certainly be interested in their performance, strengths and weaknesses. A comparison between various methods can be found in Campa et al. (1998); Coutant et al. (2000); Mc Manus (1999); Jodeau, Rockinger (2000); Syrdal (2002) and Dutta, Babel (2005). The most common criteria include goodness of fit and solution stability. The first approach is generally based on comparison between market prices and theoretical (model-implied) prices. To this end, various statistics are applied (MSE, MSPE, MAE, MAPE, RMSE).

These analyses indicate that while each method has its strengths and weaknesses, the differences in the first two moments of the implied PDF (mean and standard deviation) tend to be very small (Jackwerth 1999; Micu 2005). Should a researcher be interested in these parameters he can choose relatively freely among possible methods without significant losses in precision of the estimation.

However, higher moments of the implied distribution (skewness and kurtosis) do tend to be prone to estimation method (Mc Manus 1999). The differences among them stem from relatively high dependence between estimation procedure and implied probabilities outside the 10th and 90th percentile (Melick 1999). These extreme probabilities have in turn a significant influence on higher moments of the distribution. As reliable market data for extreme OTM options are very difficult to encounter, there are no strong grounds for deciding which method is most accurate in estimating distribution's higher moments (Cooper 1999). Taking into consideration that one cannot be sure about the extent to which implied skewness and kurtosis are influenced by the given procedure, the information content of option implied risk-neutral PDF is diminished.

In the analysis of the EUR/PLN exchange rate, the double lognormal (DLN) method is applied. The decision was based on the following criteria:

– DLN is one the most popular approaches

– some analyses (Mc Manus 1999, Syrdal 2002) indicate that while the differences between various methods in terms of goodness of fit tend to be small, the DLN approach has the best performance

– DLN is relatively easy to implement

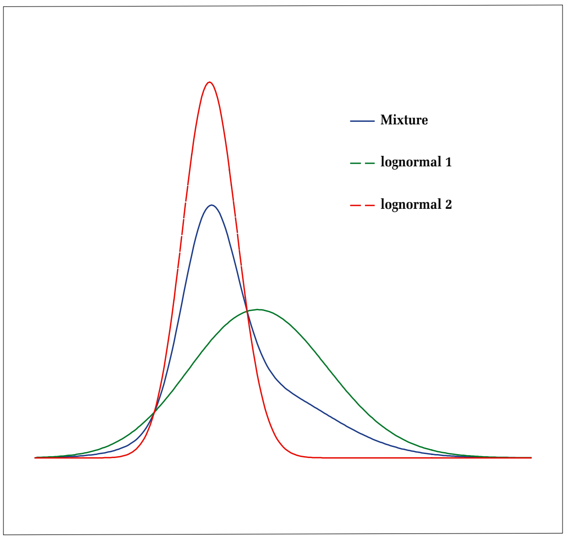
One serious drawback of the DLN method is its instability in the environment of low volatility and pronounced skewness (Cooper 1999). This problem was partly solved by imposing additional restrictions on the model (following Syrdal 2002).

5. The double lognormal approach

The method that mixes an independent lognormal distribution to obtain PDF was popularised by Melick and Thomas (1997) who applied it for PDF estimation on the oil options market. In their study, the final option implied risk-neutral PDF was a mixture of three LN distributions, still subsequent analyses used only two distributions (Bahra 1997; Söderlind, Svensson 1997; McManus 1999; Syrdal 2002; Micu 2005). The argument for such an approach is twofold. First, two LN distributions do guarantee flexibility concerning the shape of the final PDF, so that high goodness of fit is maintained. Secondly, DLN is free from problems occurring when one wants to estimate a relatively big number of parameters having limited number of option prices – as was in the original approach.

Let us recall that the DLN belongs to the second (ii) group of methods which build on the observation made by Cox and Ross (1976) that options prices under risk neutrality depend on the probability density function of the underlying asset. The idea behind these methods is to make some assumptions concerning the possible type of the final risk-neutral PDF, $q(S_{T,}\ \gamma)$, and then to estimate its parameters γ

Figure 1. *Visualisation of the double lognormal approach in the rates of return domain*



Source: Own calculations.

so that the difference between theoretical prices and market prices is minimised. Within the DLN approach, the final PDF is obtained by mixing two lognormal distributions with five parameters being estimated. These parameters are mean and standard deviation for each independent lognormal distribution (α_1, β_1 and α_2, β_2 accordingly) and weight θ attached to one of the distribution, with sum of weights equal to 1.[14] Formally, final PDF is given as:

$$q(S_T) = \theta \cdot L(S_T|\alpha_1, \beta_1) + (1-\theta) \cdot L(S_T|\alpha_2, \beta_2) \qquad (8)$$

Bahra (1997) shows that should the PDF be a product of two lognormal distributions, theoretical call (c) and put (p) option prices can be expressed analytically[15]:

$$c = e^{-r\tau}\left\{\theta\left[e^{\alpha_1+\frac{1}{2}\beta_1^2}N(d_1) - XN(d_2)\right] + (1-\theta)\left[e^{\alpha_2+\frac{1}{2}\beta_2^2}N(d_3) - XN(d_4)\right]\right\} \qquad (9)$$

$$p = e^{-r\tau}\left\{\theta\left[-e^{\alpha_1+\frac{1}{2}\beta_1^2}N(-d_1) + XN(-d_2)\right] + (1-\theta)\left[-e^{\alpha_2+\frac{1}{2}\beta_2^2}N(-d_3) + XN(-d_4)\right]\right\} \qquad (10)$$

where:

$$d_1 = \frac{-\ln(X) + \alpha_1 + \beta_1^2}{\beta_1} \qquad d_2 = d_1 - \beta_1 \qquad (11)$$

$$d_3 = \frac{-\ln(X) + \alpha_2 + \beta_2^2}{\beta_2} \qquad d_4 = d_3 - \beta_2 \qquad (12)$$

r – domestic interest rate,

τ – time to maturity (in years),

X – strike price,

$N(d)$ – standard normal distribution $N(0, 1)$.

α, β – a mean and standard deviation of each independent L-N distribution.

Moreover, mean of the final PDF must equal to the forward exchange rate – this stems from the UIP condition and assumption of risk-neutrality. Formally:

$$\theta \cdot e^{\alpha_1+\frac{1}{2}\beta_1^2} + (1-\theta)\cdot e^{\alpha_2+\frac{1}{2}\beta_2^2} = F \qquad (13)$$

The DLN aims to find five unknown parameters α_1, β_1, α_2, β_2 and θ, so that the sum of squared differences between theoretical DLN prices and market-observed prices is minimised. Formally, the problem can be written as follows:

$$\min_{\alpha_1,\alpha_2,\beta_1,\beta_2,\theta}\left\{\sum_{i=1}^{k}(c_i - c_i^*)^2 + \sum_{j=1}^{m}(p_j - p_j^*)^2 + \left[\theta\cdot e^{\alpha_1+\frac{1}{2}\beta_1^2} + (1-\theta)\cdot e^{\alpha_2+\frac{1}{2}\beta_2^2} - F\right]^2\right\} \qquad (14)$$

where:

c, p – theoretical DLN call and put option prices,

c^*, p^* – market call and put option prices,

k, m – number of available market call and put option prices with different strikes.

In order to mitigate undesired effects that the DLN can produce (local, very pronounced maximums of the PDF), additional restrictions have been put on the relation between standard deviations of the two lognormal distributions (following Syrdal 2002):

$$0{,}25 \leq \frac{\beta_1}{\beta_2} \leq 4 \qquad (15)$$

5.1. Data

Data on the implied volatility and other variables come from Bloomberg and daily observations from January 2004 to February 2007. Mid-market quotes were used. Market convention is that market makers on the foreign exchange market do not quote option prices for certain strikes but rather volatility for certain delta values. In standard situations, quotes are available for deltas corresponding to five different strike prices:

– one for zero-delta straddle strategy – a combination of call and put option with the same strike[16], with the overall forward delta of zero (premium paid in base currency)

– two for OTM call option with exercise probability at approximately 10% and 25% (10-delta[17] call and 25-delta call), with strike price above the forward rate,

– two for OTM put option with exercise probability at approximately 10% and 25% (10-delta put and 25-delta put), with strike below the forward rate,

One can see that we do not have information concerning volatility-strike space or option price-strike space. Still, using the B-S formula, one can relatively easily translate the volatility-delta space into volatility-strike space and then further into option price-strike space. In a first step, we apply the B-S formula to calculate strike prices corresponding to delta values for which implied volatility is quoted. Then, B-S is once again used to derive option prices from volatility quotes. At this point we do have all the necessary information to calculate theoretical DLN option prices and we solve the optimisation problem from the equation 14. All calculations were carried out in Microsoft Excel with Solver package.[18] While the choice of points on the volatility smile is imposed by market convention, the positive aspect is that these points cover a relatively wide spectrum of the implied PDF. Still, the number of

[14] So that the integral of the final PDF is also equal to 1.

[15] One could see that the G-K model can be viewed as a special case of DLN, where, among others, $\theta = 1$.

[16] Such a strike lies close, but it's not equal to the forward price of the underlying instrument.

[17] 10-delta in market notation describes option with delta value of 0.1; call or put further informs whether one deals with buy or sell option.

[18] Dutta, Babel (2005) also applied Solver package to solve their optimisation problems, with deliberate resignation from MATLAB.

points used in the optimisation procedure is limited to five (corresponding to approximately 10%, 25%, 50%, 75% and 90% exercise probability, with 50% being the forward rate).

As always, the quality of the data can have a significant effect on the PDF estimation. Possible problems stem from (Melick, Thomas 1997; Bliss, Panigirtzoglou 1999; Syrdal 2002):

– liquidity (probably lower for deep OTM options)

– bid-ask spread in prices of both options and underlying instrument

– not necessarily simultaneous quotes for option and underlying instrument

– errors in data of information systems

– inclusion of option prices that were not traded but only quoted.

Checking the no-arbitrage conditions eliminated some of the most obvious errors. Still, it is probable that the remaining noise in the data on the Polish market significantly exceeds that encountered on the most developed markets.

5.2. Goodness of fit

As Mean Average Percentage Error (MAPE) calculated on the whole sample does not exceed 0.75%[19], model's goodness of fit can be though as quite high. Thus, the main problems one can encounter while estimating the implied PDF stem rather from the quality of the data and risk-neutrality assumption. It seems that the assumption of risk neutrality can be of considerable importance especially on the emerging markets, i.e. due to *peso problem*, which makes them more similar to stock markets in respect to persistent skew in the implied PDF.

6. Information content of the option implied risk-neutral probability density function

There are basically two types of studies on the information content of option-implied risk-neutral PDF.

[19] Calculated with the exclusion of the forward rate, which is a more restrictive approach (forward rate tends to be many times higher than the remaining four OTM option prices)

The first group uses PDF to analyse the probabilities of abrupt changes in asset prices or, more generally, market expectations, not necessarily checking for their forecasting power. Examples of such studies have been presented earlier in this paper (Rubinstein 1994; Aït-Sahalia, Lo 1995; Melick, Thomas 1997; Leahy, Thomas 1996; Campa et al. 1997; 1998).

The second group consists of studies that verify the forecasting power of the option implied risk-neutral PDF. The main area of research within this approach concerns PDF's predictive abilities over future volatility of the underlying asset. To this end, linear regression is commonly applied (Canina, Figlewski 1993):

$$\sigma_{real} = \alpha + \beta\sigma_k + \varepsilon_t \qquad (16)$$

where σ_{real} describes realised (future) volatility, and σ_k corresponds to different measures of volatility used for forecasting. These include ATMF implied volatility (basically the B-S formula), PDF implied volatility or historical (rolling) volatility. In accordance with market convention, all volatility measures are annualised. ATMF implied volatility is directly taken from Bloomberg, whereas PDF implied volatility is calculated with a formula advocated by Jarrow and Rudd (1982):

$$\sigma_{PDF} = \sqrt{\ln(\frac{\sigma^2}{\mu^2} + 1)/\tau} \qquad (17)$$

where σ and μ correspond to PDF's standard deviation and mean, is time to maturity in years. Historical (past, rolling) 1 month volatility observed a day before the contract was sold is calculated in a standard way as a standard deviation (s) of daily foreign exchange changes divided by the square root over time to maturity:

$$\sigma_{HIST} = \frac{s}{\sqrt{\tau^*}} = \sqrt{\frac{1}{\tau^*}\frac{1}{n-1}\sum_{t=1}^{n}\left(R_t - \bar{R}\right)^2} \qquad (18)$$

τ^* is time to maturity in years calculated with respect to working days (it is assumed that there are 252 working days within a year); by annualisation we obtain τ^* = 1/252. R corresponds to a daily rate of return assuming continuous capitalisation:

$$R = \ln(\frac{EURPLN_{t+1}}{EURPLN_t}) \qquad (19)$$

Table 1. *Volatility forecasting*

$\sigma_{real} = \alpha + \beta\sigma_k + \varepsilon_t$				
σ_k	α	β	R^2	(p-value) for β
PDF	0.0163 (0.0093)	0.6875 (0.0972)	0.207	(0.000)
ATMF	0.0206 (0.009)	0.6856 (0.0970)	0.200	(0.000)
Historical 1M	0.0627 (0.0061)	0.237 (0.0635)	0.059	(0.000)

Standard errors corrected for overlapping sample using Newey-West (1987) procedure, lag truncation = 6. Asymptotic Newey-West standard errors in parentheses.
Source: Own calculations.

To apply the above regression model to the available data set, one has to take into account the overlapping sample problem. As the frequency of the observation (daily) is higher than the frequency of the data (contracts with monthly maturity), error term in the equation is subject to autocorrelation. Still, one can suspect that after some lag autocorrelation between error terms should be close to zero. Therefore, standard errors are corrected for serial correlation using the Newey-West (1987) procedure. The computations were carried out in EViews.

The results (see Table 1) indicate that the B-S implied volatility (ATMF) and PDF implied volatility provide statistically significant information concerning future volatility. Both measures are highly correlated (see Appendix) and are characterised by very similar forecasting power. While the information content of the historical (past) 1M volatility is also significant, its predictive power is considerably lower. These results do coincide with the results for more developed markets in virtually all aspects mentioned, even with regard to coefficient values (Jorion 1995; Christensen, Prabhala 1998; Weinberg 2001).

While the number of studies on the forecasting power of option implied risk-neutral PDF with respect to future volatility is relatively small, the studies on higher moments of the distribution (skewness or kurtosis) are much more scarce. The majority of works on PDFs concentrates on estimation methods and comparisons between them. Weinberg (2001) indicates that the relatively small number of studies on the PDF information content may stem from the fact that the risk-neutrality assumption does not need to correspond to real world situation. Still, should this assumption generate such a serious bias, what makes researches work on the seemingly pointless task? While this question remains without satisfactory answer, one would expect that risk-neutrality assumption could result in an estimation bias. Even though the risk-neutrality assumption accompanied many studies on the bond market, the impact of this factor seems to differ across asset classes, and could be pronounced when tail, low-probability events on the emerging or equity markets are concerned.

The BIS (1999) review indicates that one of the problems with the assessment of the PDF's information content is that PDF represents a whole range of probabilities, whereas in the end only one of these events materialises. So, as long as the realised event has non-zero occurrence probability in the previously estimated PDF, one cannot reject the viability of the PDF. This argument seems to be flawed. One could still study how the *series* of the realised asset prices corresponds to the *series* of the implied PDF and on this ground analyse whether PDF is a good gauge of real distribution or it is somehow biased. To this end, the statistics based on the empirical distribution function (Stephens 1974) can be used. Due to the fact that such a type of analysis requires relatively long series without overlapping observations, it cannot be implemented in this study.

Another obstacle for the studies on distributions' higher moments is due to the differences between methods and approaches in tail probability estimations. Tail of the distribution, where usually no data is available, may have strong impact on the skewness and kurtosis. As for now, it is hard to say which method is advisable to capture these extreme probabilities (Cooper 1999).

Bearing in mind the above mentioned problems, we will still try to assess the information content of the implied skew, foremost in terms of forecasting future exchange rate in a 1-month perspective. Let us recall that the skew in the implied distribution stems from non-zero prices of the risk-reversal (RR) contracts. RR for the EUR/PLN is persistently positive, which implies positive skew in the implied distribution. Assuming risk-neutrality, this means that the implied probability of a significant weakening of the zloty is greater than the probability of its significant strengthening. At the same time, positive skew implies that the probability mass is concentrated below the forward rate, corresponding to the higher probability of appreciation with respect to distribution's expected value. Thus, the median of the distribution lies below the forward rate. One can say that positive RR implies higher probabilities of zloty's appreciation with respect to the forward rate and, at the same time, higher probability attached to zloty's significant depreciation than to its significant appreciation. In the period under analysis, the zloty generally remained in the appreciation trend. This combination of appreciation and positive implied skew is consistent with the results obtained for other currency pairs (Malz 1997; Weinberg 2001; Campa, Chang and Reider 1998). Considering permanently positive skew in the implied PDF, one would perhaps think about the analysis that focuses on the changes in the skew.

Available studies on the predictive power of the implied skew (Weinberg 2001; Syrdal 2002) conclude that it is very small. While analysing the problem, Weinberg (2001) hints that: (i) it is difficult to say whether lack of predictive power implies low information content, (ii) risk-neutrality assumption may bias the estimation, (iii) forecasting future skew may simply be an uneasy task.

PDF estimation procedure may have a significant, possibly undesirable, impact on the distribution's tails, and consequently on the implied skew. To somehow mitigate these effects, the Pearson median skewness is used. Its advantage is that it is less sensitive to values at the ends of the distribution. On the other hand, it is not directly comparable with other skewness measures (see below). The Pearson median skewness is given as:

$$Q = \frac{3(\text{mean-median})}{\text{standard deviation}} \quad (20)$$

Table 2. *Forecasting power of the implied skewness – sign test*

Test type	No. of z = 1	Accuracy	(p-value)
Standard	24	63%	(0.14)
Modified (Qi – Qmean)	22	57%	(0.42)

Source: Own calculations.

Two types of tests are used for the analysis of the information content of the implied skew. The first one is a simple sign test of the following nature (Syrdal 2002):

$$z = 1 \begin{cases} \text{if } Q > 0 \text{ and } F - S > 0 \\ \text{or if } Q < 0 \text{ and } F - S < 0 \end{cases}$$

$$z = 0 \text{ in other cases}$$

where Q is the Pearson median skewness, F corresponds to the forward rate with maturity equal to that of the option contract and S is the future (realised) spot rate at maturity. The above test can be understood as follows. If the implied skew is positive and the spot rate lies below the forward rate, then, considering the location of the probability mass, the prediction has proven to be correct and to such event we attach value of 1. The statistical significance of the skew indications was verified with the EViews package, assuming that lack of forecasting power implies the median result of the test equal to 0.5. Given that the Newey-West (1987) procedure cannot accommodate such types of tests, the analysis was carried on the non-overlapping sample of 38 monthly observations. The persistently positive implied skew in the EUR/PLN may raise concerns over its unbiasedness as a predictor of market expectations. To partly accommodate the phenomena, a modified test is also carried out, where from each Q the mean in-the-sample skew is subtracted. In such an approach, a positive skew is deemed to be a normal situation and deviations from this 'equilibrium' are thought to provide some insight concerning market expectations.

Given the strong appreciation of the zloty in 2004, with no pronounced and persistent deprecations afterwards, the test is biased towards the rejection of the hypothesis of no forecasting power. Yet, as it was in the case of more developed markets, the results for the EUR/PLN are not very optimistic and the indications of the implied skew are not statistically significant.

Let us note that the sign test compared only the consistency between realised exchange rate changes and the indications stemming from the skew of the option-implied risk-neutral PDF. Therefore, there was no differentiation between small and big exchange rate changes. In other words, while the predictions with respect to the direction have proven to be only moderately accurate, perhaps the profit from investment strategy based on the implied skew might be still significant, yet not accommodated within the sign test. Such a type of analysis – aiming at welfare perspective – is certainly more desirable. In our case, its application is not recommended. This is due to the fact that zloty's in-sample significant appreciation combined with the persistently positive implied skew would result in an even greater bias than observed in the sign test – the mean-skew adjusted test would still be flawed.

In order to make use of the whole data set available, another test on the forecasting power of the implied skew was carried out. It is based on the following regression:

$$\ln\left(\frac{spot_{t+k}}{forward^{t}_{t+k}}\right) = \alpha + \beta\phi_i + \varepsilon_t \qquad (21)$$

where $forward^{t}_{t+k}$ is the forward rate with maturity equal to that of the option contract, $spot_{t+k}$ is the future (realised) spot rate at maturity and ϕ_i corresponds to different measures of the skew of the option implied risk-neutral PDF. The test logic is thus somewhat similar to that of the sign test; still the magnitude of the exchange rate changes is allowed to vary along with skewness measures. The overlapping sample problem was solved by applying the Newey-West (1987) procedure. The calculations were carried out in EViews. The Pearson median skewness was kept as a basic measure of the skew, with some adjustment. The adjustment consisted of subtracting from each Q the skew of the lognormal distribution with standard deviation equal to the volatility in the B-S formula (ATMF). The idea behind is that the B-S model assumes positive skew in the distribution of the underlying instruments' price (and zero skew in rates of return). One would expect that the B-S implied skew should not have any forecasting power, so it is used to adjust the PDF implied skew. The skew of the B-S distribution is given by the standard formula for lognormal distribution:

$$skew = (e^{\sigma^2} + 2) \times \sqrt{e^{\sigma^2} - 1} \qquad (22)$$

where ATMF volatility is used as a measure of standard deviation σ. Still, as it was mentioned earlier in the text, the above and Pearson's measures of skew are not directly comparable, so the test possibly includes some unknown bias. Thus, a simpler form of the test is also carried out, with the Pearson median skewness adjusted by the sample mean Q.[20]

[20] Within this framework, there is virtually no difference between such an approach and one based on the unadjusted Pearson median skewness – the only thing that changes is the intercept (by the value of the mean skew).

Table 3. *Forecasting power of the implied skewness - regression*

$\ln(\frac{spot_{t+k}}{forward^{t}_{t+k}}) = \alpha + \beta\phi_i + \varepsilon_t$				
ϕ_i	α	β	R^2	(p-value) for β
SkewPearson PDF – SkewATMF	-0.0075 (0.0021)	-0.0095 (0.0151)	0.003	(0.53)
Pearson PDF	-0.0077 (0.0020)	-0.0263 (0.0199)	0.012	(0.19)

Standard errors corrected for overlapping sample using Newey-West (1987) procedure, lag truncation = 6. Asymptotic Newey-West standard errors in parentheses.
Source: Own calculations.

While the β coefficients have the expected sign (based on the distribution of the probability mass), they are not statistically different from zero. Intercepts are found to be statistically significant most probably due to the pronounced and consistent appreciation of the zloty within the sample. The results coincide with earlier observations from the sign test.

Concluding, it seems that there are no grounds for using the skew of the option implied risk-neutral PDF for forecasting the EUR/PLN. It seems that the implied PDF could rather be used as an approximation of market situation, not necessarily informing about market expectations.

7. Conclusions

Due to their substantial information content, option-implied risk-neutral PDFs have become an increasingly popular target of scientific analysis. The prices of financial instruments reflect all (probability weighted) market-possible scenarios, yet in most of the cases it is extremely difficult, if not impossible, to derive probabilities attached to realisation of particular events. The prime advantage of the option market is that through PDF analysis one can approximate the implied probability of any event. Since option prices on the market do not seem to coincide with the Black-Scholes assumption of lognormal distribution of the underlying asset, an analysis aiming at estimating how the market-implied risk-neutral PDF really looks like is even more interesting. Still, the number of liquid options across the strike space (underlying instruments' price) is far from dense. Thus, estimation techniques generally tend to optimise some conditions to find the PDF that best fits the data.

Most of the research has concentrated on developing, testing and comparing characteristics of various estimation techniques, whereas less attention has been paid to the analysis of their information content and forecasting power. This paper went in the latter direction and investigated the forecasting power of the EUR/PLN 1 month options. To this end, double lognormal approach was applied.

Volatility forecasted based on the PDF analysis has proven to provide better information concerning future volatility than historical volatility. Yet PDF implied volatility is highly correlated with the volatility in the Black-Scholes model (ATMF), with very similar forecasting power. As it is the case for more developed markets, there are no grounds for using the skew of the option-implied risk-neutral PDF for forecasting the zloty exchange rate. Low information content does not directly follows as estimated PDF could be used as an approximation of market situation, not necessarily informing about market expectations. Moreover, the quality of the data, an unknown relation between applied estimation technique and tail probabilities together with the accompanying assumption of risk neutrality suggest caution in drawing conclusions from available estimations, especially concerning higher moments of the distribution.

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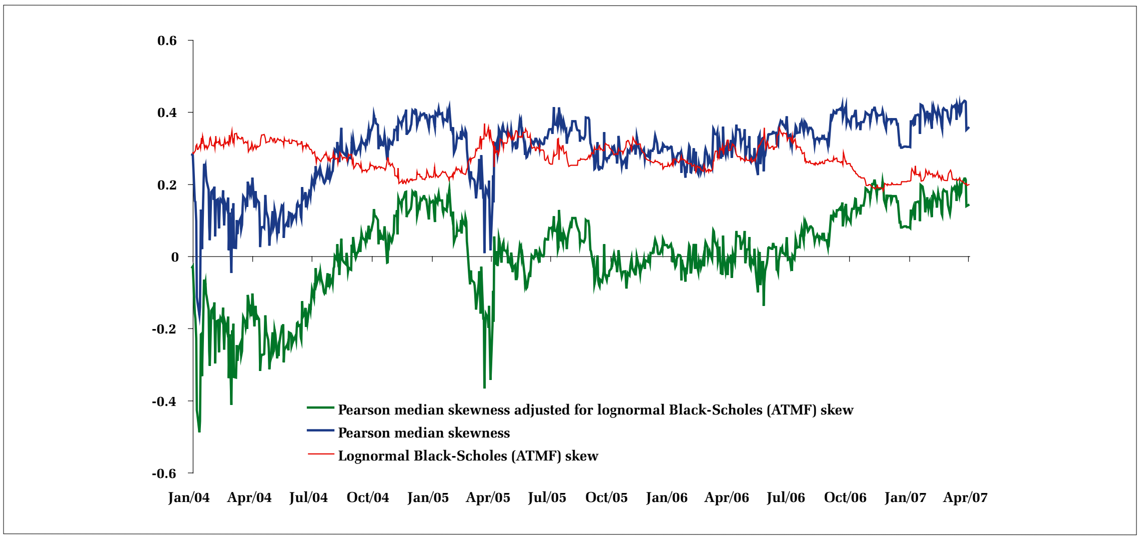
Appendix

Table A1. *Correlation of volatility measures (daily data)*

	PDF	ATMF	Historical
PDF	1	0.979	0.663
ATMF		1	0.660
Historical			1

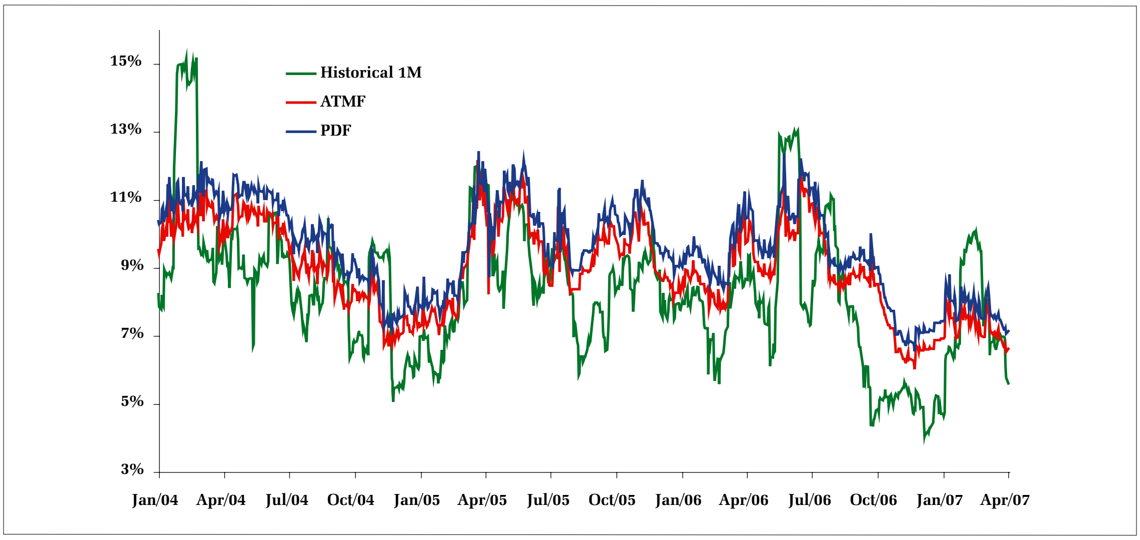
Source: Own calculations.

Figure A1. *Skewness measures*



Source: Own calculations.

Figure A2. *Volatility measures*



Source: Own calculations.